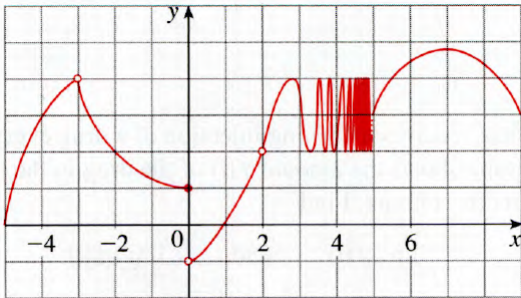


科目名稱: 微積分 (上)(3學分)  
 考試時間: 10 月 3 日第二節  
 \* (每題 7 分, 滿分 105 分)

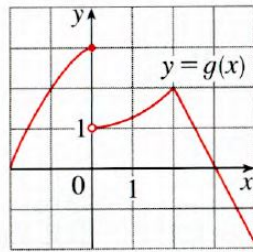
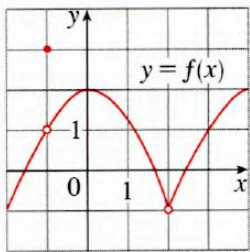
1. Use the graph of  $f$  to state the value of each quantity, if it exists. If the limit does not exist, explain why.

- (a)  $\lim_{x \rightarrow -3^-} f(x)$ .    (b)  $\lim_{x \rightarrow -3^+} f(x)$ .    (c)  $\lim_{x \rightarrow -3} f(x)$ .  
 (d)  $\lim_{x \rightarrow 0^-} f(x)$ .    (e)  $\lim_{x \rightarrow 0^+} f(x)$ .    (f)  $\lim_{x \rightarrow 0} f(x)$ .    (g)  $\lim_{x \rightarrow 5} f(x)$ .



2. Use the graphs of  $f$  and  $g$  to evaluate each limit, if it exists. If the limit does not exist, explain why.

- (a)  $\lim_{x \rightarrow 2} f(x)$ .    (b)  $\lim_{x \rightarrow 2} g(x)$ .    (c)  $\lim_{x \rightarrow 0} f(x)$ .    (d)  $\lim_{x \rightarrow 0} g(x)$ .  
 (e)  $\lim_{x \rightarrow 2} [f(x) + g(x)]$ .    (f)  $\lim_{x \rightarrow 0} [x^2 f(x)]$ .    (g)  $\lim_{x \rightarrow 3} \frac{f(x)}{g(x)}$ .



3. Evaluate  $\lim_{t \rightarrow -3} \frac{t^2 - 9}{2t^2 + 7t + 3}$ .

4. Evaluate  $\lim_{h \rightarrow 0} \frac{(3 + h)^{-1} - 3^{-1}}{h}$ .

5. Prove that  $\lim_{x \rightarrow 0} x^2 \cos \frac{1}{x} = 0$ .

6. Let  $g(x) = \frac{x^2 + x - 6}{|x - 2|}$ .

Find (a)  $\lim_{x \rightarrow 2^+} g(x)$ . (b)  $\lim_{x \rightarrow 2^-} g(x)$ . (c) Does  $\lim_{x \rightarrow 2} g(x)$  exist? (Explain why).

7. Show that  $f(x) = |x|$  is continuous at 0.

8. Show that  $f(x) = \lfloor x \rfloor$  is not continuous but is continuous from the right at 0.

9. Evaluate  $\lim_{x \rightarrow \pi} \cos(x + \sin x)$ .

10. Find the values of  $a$  and  $b$  that make  $f$  continuous everywhere.

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x < 2, \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3. \\ 2x - a + b & \text{if } x \geq 3. \end{cases}$$

11. Use the Intermediate Value theorem to show that there is a root of the equation  $x^4 + x = 3$  between 1 and 2.

12. Let  $f(x) = x^2$ . Use the definition of derivative to find  $f'(x)$ .

13. Show that  $f(x) = |x|$  is not differentiable at 0.

14. (a) Sketch the graph of the function  $f(x) = x|x|$ .

(b) Prove that  $f$  is differentiable at 0.

(c) Prove that  $f'(x) = 2|x|$ . (Hint: You may use the result of problem 12.)

15. If  $f(x) = \lfloor x \rfloor + \lfloor -x \rfloor$ , show that  $f(x)$  is not continuous at 2.

題號	答案	來源
1	(a)4 (b)4 (c)4 (d)1 (e) - 1 (f)The limit doesn't exist (g)The limit doesn't exist	1.5 - 習題 6
2	(a) - 1 (b)2 (c)2 (d)The limit doesn't exist (e)1 (f)0 (g)The limit doesn't exist	1.6 - 習題 2
3	$\frac{6}{5}$	1.6 - 習題 15
4	$\frac{-1}{9}$	1.6 - 習題 24
5	略	1.6 - 例題 11*
6	(a)5 (b) - 5 (c)The limit doesn't exist	1.6 - 習題 49
7	略	1.6 - 例題 7
8	略	1.8 - 例題 3*
9	- 1	1.8 - 習題 36*
10	$a = \frac{1}{2}, b = \frac{1}{2}$	1.8 - 習題 46
11	略	1.8 - 習題 53
12	$2x$	2.2 - 例題 2*
13	略	2.2 - 例題 5*
14	(a)略 (b)0 (c) $2 x $	2.2 - 習題 59
15	略	1.6 - 習題 12*

\* 為非勾選習題、類似題。  
證明題略過。