

科目名稱: 微積分 (上)(3學分)

考試時間: 12 月 12 日第二節

* (每題 7 分, 滿分 105 分)

1. Show that $x^3 + 3x - \sin x = 0$ has at most one root.
2. Use the Mean Value Theorem to prove the inequality $|\sin a - \sin b| \leq |a - b|$ for all a and b .
3. Find the local maximum and minimum values of $f(x) = \frac{x^2}{x-1}$ using the First Derivative Test.
4. $F(x) = x\sqrt{6-x}$.
 - (a) Find the intervals of increase or decrease.
 - (b) Find the intervals of concavity and the inflection points.
5. Find the horizontal asymptotes of the graph of the function $f(x) = \frac{\sqrt{x^2-5}}{x+1}$.
6. Evaluate $\lim_{x \rightarrow \infty} (\sqrt{9x^2+x} - 3x)$.
7. Evaluate $\int_0^2 |2x-1| dx$.
8. Evaluate $\int_1^4 \sqrt{t}(1+t) dt$.
9. Evaluate $\int \left(\frac{\cos \theta}{\sin^2 \theta} - 2 \sec^2 \theta \right) d\theta$.
10. Find a function f and a number a such that $6 + \int_a^x \frac{f(t)}{t^2} dt = 2\sqrt{x}$ for all $x > 0$.
11. Evaluate $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i^4}{n^5} + \frac{i}{n^2} \right)$.
12. Evaluate $\frac{d}{dx} \int_{\sqrt{x}}^0 \cos(t^2) dt$.
13. Evaluate $\int_0^2 (x + \sqrt{4-x^2}) dx$.
14. Evaluate $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \sqrt{1 - \left(\frac{i}{n} \right)^2}$.

15. Sketch the graph of a function satisfies all of the given conditions.

(a) $\lim_{x \rightarrow \pm\infty} f(x) = 2$, $\lim_{x \rightarrow 1^+} f(x) = \infty$, $\lim_{x \rightarrow 1^-} f(x) = -\infty$, $\lim_{x \rightarrow -1^+} f(x) = -\infty$, $\lim_{x \rightarrow -1^-} f(x) = \infty$.

(b) $f'(x) > 0$ when $x < 0$ ($x \neq -1$), $f'(x) < 0$ when $x > 0$ ($x \neq 1$).

(c) $x = 0$ is the only critical number $f(0) = 0$.

(d) $f''(x) < 0$ when $|x| < 1$, $f''(x) > 0$ when $|x| > 1$.

題號	答案	來源
1	略	3.2 – 例題 2*
2	略	3.2 – 習題 31
3	The local max. is $f(0) = 0$, the local min. is $f(2) = 4$	3.3 – 習題 16
4	(a) F is increasing on $(-\infty, 4]$, decreasing on $[4, 6]$ (b) F is concave downward on $(-\infty, 6)$, no inflection point.	3.3 – 習題 39
5	$y = 1$ and $y = -1$	3.4 – 例題 4*
6	$\frac{1}{6}$	3.4 – 習題 21
7	$\frac{5}{2}$	4.4 – 習題 40
8	$\frac{256}{15}$	4.4 – 習題 31
9	$-\csc \theta - 2 \tan \theta + C$	4.4 – 例題 1,2*
10	$f(x) = x^{\frac{3}{2}}, a = 9$	4.3 – 習題 75
11	$\frac{7}{10}$	4.3 – 習題 67
12	$-\cos x \cdot \frac{1}{2\sqrt{x}}$	4.3 – 習題 55*
13	$2 + \pi$	4.2 – 例題 *
14	$\frac{\pi}{4}$	4.2 – 習題 74*
15	略	3.5 – 例題 1

* 為非勾選習題、類似題、證明題、圖形題略過。