

科目名稱: 微積分 (上)(3學分)

考試時間: 1 月 9 日第二節

* (每題 7 分, 滿分 105 分)

1. Evaluate $\int x^5 \sqrt{1+x^2} dx$.

2. Evaluate $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$.

3. Evaluate $\int_0^a x \sqrt{a^2 - x^2} dx$.

4. Evaluate $\int_0^1 \frac{1}{(1+\sqrt{x})^4} dx$.

5. If f is continuous and $\int_0^4 f(x) dx = 10$, find $\int_0^2 f(2x) dx$.

6. Find the inverse function of $f(x) = \frac{4x-1}{2x+3}$.

7. Find $(f^{-1})'(3)$ if $f(x) = 3 + x^2 + \tan\left(\frac{\pi x}{2}\right)$.

8. If $f(x) = \int_3^x \sqrt{1+t^3} dt$, find $(f^{-1})'(0)$.

9. If $h(x) = x + \sqrt{x}$, find $h^{-1}(6)$.

10. Suppose f^{-1} is the inverse function of a differentiable function f and $f(4) = 5, f'(4) = \frac{2}{3}$. Find $(f^{-1})'(5)$.

11. Prove that $\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$.

12. Evaluate $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$.

13. Differentiate $y = \sin^{-1}(2x+1)$.

14. Evaluate $\int_0^{\frac{\sqrt{3}}{4}} \frac{1}{1+16x^2} dx$.

15. Evaluate $\int \frac{t^2}{\sqrt{1-t^6}} dt$.

題號	答案	來源
1	$\frac{1}{7}(1+x^2)^{\frac{7}{2}} - \frac{2}{5}(1+x^2)^{\frac{5}{2}} + \frac{1}{3}(1+x^2)^{\frac{3}{2}} + C$	4.5 - 例題 5
2	$-2 \cos \sqrt{x} + C$	4.5 - 例題 16
3	$\frac{1}{3}a^3$	4.5 - 習題 44
4	$\frac{1}{6}$	4.5 - 習題 51
5	5	4.5 - 習題 59
6	$\frac{3x+1}{4-2x}$	6.1 - 習題 24
7	$\frac{2}{\pi}$	6.1 - 習題 41
8	$\frac{1}{\sqrt{28}}$	6.1 - 習題 45
9	4	6.1 - 習題 19
10	$\frac{3}{2}$	6.1 - 習題 43
11	略	6.6 - 例題 *
12	$\frac{2}{\sqrt{5}}$	6.6 - 習題 7
13	$y' = \frac{1}{\sqrt{-x^2-x}}$	6.6 - 習題 25
14	$\frac{\pi}{12}$	6.6 - 習題 62
15	$\frac{1}{3} \sin^{-1}(t^3) + C$	6.6 - 習題 67

* 為非勾選習題、類似題。
證明題、圖形題略過。