中原大學　107　學年度　□上學期 ■下學期　考試命題紙 ■第二次會考

科目名稱: 微積分 (下)(3學分)
考試時間: 5 月 15 日第二節
* ⟨每題 7 分, 滿分 105 分⟩

1. (a) If $z = f(x, y)$, define $f_y(x, y)$ in terms of limit.
   (b) Use (a) to find $f_y(x, y)$ if $f(x, y) = x^2 e^y$.

2. If $f(x, y) = \cos\left(\dfrac{y}{1+x}\right)$, calculate f_{xx}, f_{xy} and f_{yy}.

3. Let $z = f(x, y)$ and $u =< a, b >$ be a unit vector.
   (a) Define $D_u f(x, y)$, the directional derivative of f in the direction of u.
   (b) Define $\nabla f(x, y)$, the gradient of f.
   (c) Express $D_u f(x, y)$ in terms of $\nabla f(x, y)$.

4. If $f(x, y, z) = x \sin yz$, find
   (a) the gradient of f.
   (b) the directional derivative of f at $(3, 0, 1)$ in the direction of $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

5. Suppose that the temperature at a point (x, y, z) in space is given by $T(x, y, z) = 80/(1 + x^2 + 2y^2 + 3z^2)$, where T is measured in degrees Celsius and x, y, z in meters.
   (a) In which direction does the temperature increase fastest at the point $(1, -1, 2)$.
   (b) What is the maximum rate of increase ?

6. Find the directional derivative of $f(x, y) = x^2 e^{-y}$ at the point $(-2, 0)$ in the direction toward the point $(2, -3)$.

7. At what points does the normal line through the point $(1, 2, 1)$ on the ellipsoid $4x^2+y^2+4z^2 = 12$ intersect the sphere $x^2 + y^2 + z^2 = 102$?

8. If $z = x^2 y + 3xy^4$, where $x = \sin(2t)$ and $y = \cos(t)$, find $\dfrac{dz}{dt}$ when $t = 0$.

9. If $u = x^4 y + y^2 z^3$, where $x = rse^t, y = rs^2 e^{-t}$ and $z = r^2 s \sin(t)$, find the value of $\dfrac{\partial u}{\partial s}$ when $r = 2, s = 1, t = 0$.

10. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $x^3 + y^3 + z^3 + 6xyz = 1$.

11. If $z = f(x, y)$ where $x = s + t$ and $y = s - t$, show that $\left(\dfrac{\partial z}{\partial x}\right)^2 - \left(\dfrac{\partial z}{\partial y}\right)^2 = \dfrac{\partial z}{\partial s}\dfrac{\partial z}{\partial t}$.

12. Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

13. Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.

14. Find three positive numbers whose sum is 100 and whose product is a maximum.

15. Find the dimensions of the rectangular box with largest volume if the total surface area is given as 64 cm^2.

107 學年度第二學期理工電資學院微積分 (3學分) 第二次會考答案 2019.5.15

題號	答案	來源
1	x^2e^y	14.3 – 習題 45*
2	$f_{xx} = \frac{-2y}{(1+x)^3}\sin(\frac{y}{1+x}) - \frac{y^2}{(1+x)^4}\cos(\frac{y}{1+x}), f_{yy} = -\frac{1}{(1+x)^2}\cos(\frac{y}{1+x})$ $f_{xy} = \frac{1}{(1+x)^2}\sin(\frac{y}{1+x}) + \frac{y}{(1+x)^3}\cos(\frac{y}{1+x})$	14.3 – 例題 4*
3	$(a)\lim_{h\to 0}\frac{f(x+ha, y+hb) - f(x,y)}{h},\ (b)\frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j},\ (c)f_x(x,y)a + f_y(x,y)b$	14.6 – 定義
4	$(a)\sin(yz)\mathbf{i} + xz\cos(yz)\mathbf{j} + xy\cos(yz)\mathbf{k},\ (b) - 2$	14.6 – 例題 5*
5	$(a)\frac{5}{8} < -1, 2, -6 >, (b)\frac{5}{8}\sqrt{41}$	14.6 – 例題 7*
6	$-\frac{4}{5}$	Review – 習題 45
7	$t = -1, (x,y,z) = (-7,-2,-7); t = \frac{2}{3}, (x,y,z) = (\frac{19}{3}, \frac{14}{3}, \frac{19}{3})$	14.6 – 習題 60
8	6	14.5 – 例題 1
9	192	14.5 – 例題 5
10	$\frac{\partial z}{\partial x} = -(\frac{x^2+2yz}{z^2+2xy}), \frac{\partial z}{\partial y} = -(\frac{y^2+2xz}{z^2+2xy})$	14.5 – 例題 9
11	略	14.5 – 習題 46
12	The local min. $f(-1,-1) = -1, f(1,1) = -1$, critical points : $(0,0), (-1,-1), (1,1)$	14.7 – 例題 3
13	$\frac{5\sqrt{6}}{6}$	14.7 – 例題 5
14	$(x,y,z) = (\frac{100}{3}, \frac{100}{3}, \frac{100}{3})$	14.7 – 習題 45
15	$(x,y,z) = (\frac{8}{\sqrt{6}}, \frac{8}{\sqrt{6}}, \frac{8}{\sqrt{6}})$	14.7 – 習題 50

* 爲非勾選習題、類似題.
證明題、圖形題略過.