

科目名稱: 微積分 (下)(3學分)

考試時間: 6 月 19 日第二節

\* (每題 7 分, 滿分 105 分)

1. Find the 6th Taylor polynomials at  $\frac{\pi}{2}$  for  $f(x) = \sin x$ .
2. Find the binomial series for  $f(x) = \frac{1}{\sqrt{1+x}}$ .
3. Find the Maclarin series for  $f(x) = \tan^{-1} x$ .
4. If  $R = \{(x, y) | -1 \leq x \leq 1, -2 \leq y \leq 2\}$ , evaluate the integral  $\iint_R \sqrt{1-x^2} dA$ .
5. Evaluate the double integral  $\iint_R (x - 3y^2) dA$ , where  $R = \{(x, y) | 0 \leq x \leq 2, 1 \leq y \leq 2\}$ .
6. Evaluate  $\iint_R y \sin(xy) dA$ , where  $R = [1, 2] \times [0, \pi]$ .
7. Find the volume of the solid that lies under the plane  $4x + 6y - 2z + 15 = 0$  and above the rectangle  $R = \{(x, y) | -1 \leq x \leq 2, -1 \leq y \leq 1\}$ .
8. Evaluate  $\iint_D (x + 2y) dA$ , where  $D$  is the region bounded by the parabolas  $y = 2x^2$  and  $y = 1 + x^2$ .
9. Use a double integral to find the volume of the solid bounded by the coordinate planes and the plane  $3x + 2y + z = 6$ .
10. Evaluate the iterated integral  $\int_0^1 \int_x^1 \sin(y^2) dydx$ .
11. Use a double integral to find the area bounded by the line  $y = x - 1$  and the parabola  $y^2 = 2x + 6$ .
12. Evaluate  $\iint_R (3x + 4y^2) dA$ , where  $R$  is the region in the upper half-plane bounded by the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$ .
13. Find the volume of the solid bounded by the plane  $z = 0$  and the paraboloid  $z = 1 - x^2 - y^2$ .

14. Use a double integral to find the area enclosed by one loop of the four-leaved rose  $r = \cos 2\theta$ .

15. Convert the double integral  $\int_0^{\frac{1}{2}} \int_{\sqrt{3}y}^{\sqrt{1-y^2}} f(x, y) \, dx dy$  to polar coordinates.

題號	答案	來源
1	$1 - \frac{1}{2}(x - \frac{\pi}{2})^2 + \frac{1}{24}(x - \frac{\pi}{2})^4 - \frac{1}{720}(x - \frac{\pi}{2})^6$	講義
2	$1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \dots$	講義
3	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	講義
4	$2\pi$	15.1 - 例題 2
5	$-12$	15.1 - 例題 5
6	$0$	15.1 - 例題 6
7	$51$	15.1 - 習題 37
8	$\frac{32}{15}$	15.2 - 例題 1
9	$6$	15.2 - 習題 29*
10	$\frac{1}{2}(1 - \cos 1)$	15.2 - 例題 5
11	$18$	15.2 - 例題 3*
12	$\frac{15}{2}\pi$	15.3 - 例題 1
13	$\frac{\pi}{2}$	15.3 - 例題 2
14	$\frac{\pi}{8}$	15.3 - 例題 3
15	$\int_0^{\frac{\pi}{6}} \int_0^1 f(r \cos \theta, r \sin \theta) r dr d\theta$	15.3 - 習題 31

\* 為非勾選習題、類似題。  
證明題、圖形題略過。