中原大學 108 學年度 ■上學期 □下學期 考試命題紙 ■ 第二次會考

科目名稱: 微積分 (上)(3 學分)
考試時間: 12 月 11 日第二節

I. 填充題. (45 分)

1. State the Mean Value Theorem. Let f be a function that satisfies the following hypotheses ：
   (a) f is continuous on the closed interval $[a, b]$.
   (b) f is differentiable on the open interval (a, b).
   Then there is a number c in (a, b) such that $f'(c) = \underline{\dfrac{f(b)-f(a)}{b-a}}$. §3.2 定理

2. Given $f(x) = x^{\frac{2}{3}}(6-x)^{\frac{1}{3}}$, $f'(x) = \dfrac{4-x}{x^{\frac{1}{3}}(6-x)^{\frac{2}{3}}}$, $f''(x) = \dfrac{-8}{x^{\frac{4}{3}}(6-x)^{\frac{5}{3}}}$, the inflection point is $\underline{(6,0)}$. §3.3 eg7

3. Consider $f(x) = 3x^4 - 4x^3 - 12x^2 - 5$. The local maximum value is $\underline{-5}$. §3.3 eg1*

4. Consider the curve $y = f(x) = x^4 - 4x^3$. Find the intervals where f is concave downward ： $\underline{(0,2)}$. §3.3 eg6

5. Find $\lim\limits_{x\to\infty} (\sqrt{9x^2+x} - 3x) = \underline{\dfrac{1}{6}}$. §3.4 ex21

6. The equation of the slant asymptote for $y = \dfrac{x^3}{(x+1)^2}$ is $\underline{y = x - 2}$. §3.5 ex52

7. The expression of the limit $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{1}{n + \frac{i^2}{n}}$ as an integral on the interval $[0, 1]$ is $\underline{\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx}$. §4.2 ex74

8. The curve $y = \displaystyle\int_0^x \frac{t^2}{t^2+t+2}\,dt$ is concave upward on the interval ： $\underline{(-\infty,-4)\cup(0,\infty)}$. §4.3 ex59*

9. If $f(x) = \displaystyle\int_0^{\cos x} \sqrt{1+t^2}\,dt$ and $g(y) = \displaystyle\int_3^y f(x)\,dx$, then $g''(\frac{\pi}{6}) = \underline{-\dfrac{\sqrt{7}}{4}}$. §4.3 ex62*

II. 計算、證明題. (60 分)

1. Use the Intermediate Value Theorem and the Mean Value Theorem to show that the equation $x^3 + x - 1 = 0$ has exactly one root.

2. Evaluate (a) $\lim_{x\to\infty} \sin\frac{1}{x}$. (b) $\lim_{x\to\infty} \sqrt{x}\sin\frac{1}{x}$.

3. $f(x) = x^4 - 2x^2 + 3$.
(a) Find the intervals on which f is increasing or decreasing. (b) Find the intervals of concavity.

4. Use the guidelines of Section 3.5 to sketch the graph of $f(x) = \dfrac{x^2}{\sqrt{x+1}}$.
$\left(\text{Hint: } f''(x) = \dfrac{3x^2+8x+8}{4(x+1)^{\frac{5}{2}}}\right)$

5. Evaluate the limit of the Riemann sum as $n \to \infty$ for $f(x) = x^3 - 6x$, taking the sample points to be right endpoints and $a = 0, b = 3$. $\left(\text{Hint: } \sum_{i=1}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2\right)$

6. Evaluate $\int_0^{10} |x-5|\,dx$.

108 學年度第一學期理工電資學院微積分 (3 學分) 第二次會考答案 2019.12.11

題號	答案	來源
1	略	3.2 – 例題 2
2	(a) 0 (b) 0	3.4 – (a) 例題 6, (b) 習題 32
3	(a) f is increasing on $(-1,0)\cup(1,\infty)$, f is decreasing on $(-\infty,-1)\cup(0,1)$. (b) f is concave upward on $(-\infty,\frac{\sqrt{3}}{3})\cup(\frac{\sqrt{3}}{3},\infty)$, f is concave downward on $(-\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3})$.	3.3 – 習題 11
4	略	3.5 – 例題 2
5	$-\frac{27}{4}$	4.2 – 例題 2
6	25	4.2 – 習題 40*

* 為非勾選習題、類似題.