

科目名稱：微積分(上)(3 學分)

考試時間：10 月 9 日 第二節

* 第一部分填充題每格 5 分，第二部分計算證明題每題 10 分，滿分 105 分

I. Fill in the blank. (45%)

- Let $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+9x+9}-3}{x} = \frac{a}{2}$. Then $a = \underline{3}$ 。
- Let $f(x) = \frac{|x-1|}{x-1}$. Then $\lim_{x \rightarrow 1^-} f(x) = \underline{-1}$ and $\lim_{x \rightarrow 1^+} f(x) = \underline{1}$ 。
- Find the limit $\lim_{x \rightarrow 3^-} \frac{\sqrt{x}}{(x-3)^5} = \underline{-\infty}$ (or 不存在)。
- Let $f(x) = \frac{x^3-2x^2-2x-3}{x^3-4x^2+4x-3}$ for $x \neq 3$. Then $f(3)$ should be defined to be $f(3) = \underline{13/7}$ so that it becomes continuous everywhere.
- Suppose that the tangent line to the graph of a function f at $x = 1$ passes through the point $(4,9)$ and that $f(1) = 3$. Then $f'(1) = \underline{2}$ 。
- Let $f(x) = ax^2 + bx + c$ for all x , $f(2) = 26$, $f'(2) = 23$, and $f''(2) = 14$. Then $f(1) = \underline{10}$ 。
- Let $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$, and f be differentiable everywhere. Then $m = \underline{4}$ and $b = \underline{-4}$ 。

II. Calculation and Proof. (60%)

1. Show that $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0$.
2. Show that there is a root of the equation $4x^3 - 6x^2 + 3x - 2 = 0$ between 1 and 2.
3. Let $f(x) = \sqrt{x}$. Find $f'(x)$ for $x > 0$ by using the definition of derivative.
4. Find equations of the tangent line and normal line to the curve $y = \frac{\sqrt{x}}{1+x^2}$ at the point $(1, \frac{1}{2})$.
5. Differentiate.
 - (a) $f(x) = \left(\frac{1}{x^2} - \frac{1}{x^4}\right)(x + 5x^3)$.
 - (b) $y = \frac{x^2+1}{x^3-1}$.
6. Find an equation of the tangent line to the curve $y = x^4 + 1$ that is parallel to the line $32x - y = 15$.

題號	答案	來源
1	略	1.6 - 例題 11
2	略	1.8 - 例題 9
3	$\frac{1}{2\sqrt{x}}$	2.2 - 例題 3
4	The tangent line is $y - \frac{1}{2} = -\frac{1}{4}(x - 1)$; the normal line is $y - \frac{1}{2} = 4(x - 1)$.	2.3 - 例題 12
5	(a) $f'(x) = \left(\frac{-2}{x^3} + \frac{4}{x^5}\right)(x + 5x^3) + \left(\frac{1}{x^2} - \frac{1}{x^4}\right)(1 + 15x^2)$. (b) $y' = \frac{2x(x^3-1)-3x^2(x^2+1)}{(x^3-1)^2}$.	2.3 - 習題 27*、29
6	The tangent line is $y - 17 = 32(x - 2)$.	2.3 - 習題 80