中原大學 108 學年度 ■上學期 □下學期 考試命題紙 ■期中考

科目名稱: 微積分 (上)(3 學分)
考試時間: 11 月 6 日第二節

I. 填充題. (45 分)

1. The $53th$ derivative of $\cos x$ is $\underline{-\sin x}$. (§2.4 ex4*)

2. Find $\lim_{\theta\to 0}\dfrac{\sin\theta}{\theta+\tan\theta} = \underline{\dfrac{1}{2}}$. (§2.4 exe 45)

3. Find $\lim_{x\to 0} x\cot x = \underline{1}$. (§2.4 ex 6)

4. If $F(x) = f(3f(4f(x)))$ with $f(0) = 0$ and $f'(0) = 2$, then $F'(0) = \underline{96}$. (§2.5 exe71)

5. If $\sin(xy) = \cos(x+y)$, then $y' = \underline{\dfrac{-\sin(x+y) - y\cos(xy)}{x\cos(xy) + \sin(x+y)}}$. (§2.6 exe 19)

6. If $f(x) = \tan(\sec(\sin x))$, find $f'(x) = \underline{\sec^2(\sec(\sin x))\sec(\sin x)\tan(\sin x)\cos x}$. (§2.5 exe 39)

7. $\tan 2^\circ \approx \underline{\dfrac{\pi}{90}}$. (§2.9 exe27)

8. The absolute maximum value of $f(x) = 2\cos t + \sin 2t$ on the interval $[0, \frac{\pi}{2}]$ is $\underline{\dfrac{3\sqrt{3}}{2}}$. (§3.1 exe55)

9. If $y = \sqrt{3+x^2}$, $x = 1$ and $dx = -0.1$, find the differential $dy = \underline{-0.05}$. (§2.9 exe17)

II. 計算、證明題. (60 分)

1. (a) Prove that $\dfrac{d}{dx}\tan x = \sec^2 x$. (b) Find y'' if $x^4 + y^4 = 16$ (不須化簡).

2. Find the linearization of the function $f(x) = \sqrt{x+3}$ at $a = 1$ and use it to approximate the number $\sqrt{3.98}$.

3. Differentiate $f(x) = \dfrac{\sec x}{1+\tan x}$. For what values of x does the graph of f have a horizontal tangent ?

4. Differentiate (a) $y = \sqrt{\sec(x^3)}$. (b) $y = \cos^4(\sin^3 x)$.

5. (a) Find y' if $x^3 + y^3 = 6xy$.
(b) Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point $(3, 3)$.
(c) At what point in the first quadrant is the tangent line horizontal ?

6. Find the critical numbers of (a) $f(x) = x^{\frac{3}{5}}(4-x)$. (b) $f(x) = 4x - \tan x$.

108 學年度第一學期理工電資學院微積分 (3 學分) 期中考答案 2019.11.6

題號	答案	來源
1	(a)略　　$(b)-\dfrac{3x^2y^3-3x^3y^2(\frac{-x^3}{y^3})}{y^6}$	$(a)2.4-$ 習題 18 $(b)2.6-$ 例題 4
2	1.995	2.9 – 例題 1
3	$x=n\pi+\dfrac{\pi}{4}, n\in N$	2.4 – 例題 2
4	$(a)\dfrac{3}{2\sqrt{\sec(x^3)}}\sec(x^3)\tan(x^3)x^2$ $(b)-12\cos^3(\sin^3 x)\sin(\sin^3 x)\sin^2 x\cos x$	2.5 – 例題 8, 習題 44
5	$(a)y'=\dfrac{2y-x^2}{y^2-2x}$ (b)The tangent line is $y-3=-(x-3)$ $(c)x=2^{\frac{4}{3}}, y=2^{\frac{5}{3}}$	2.6 – 例題 2
6	$(a)x=\dfrac{3}{2}, 0$　$(b)x=\pm\dfrac{\pi}{3}+n\pi, n\in N$	3.1 – 例題 7, 習題 40

* 為非勾選習題、類似題.