中原大學 108 學年度 ■上學期 □下學期 考試命題紙 ■期末考

科目名稱: 微積分 (上)(3 學分)
考試時間: 01 月 08 日第二節

I. 填充題. (45 分)

1. Evaluate $\int x(3x^2-9)^9\,dx =$ $\frac{1}{60}(3x^2-9)^{10}+C$

2. Evaluate $\int_0^1 \frac{1}{(1+\sqrt{x})^4}\,dx =$ $\frac{1}{6}$

3. Find $\int_0^1 x^5\sqrt{1+x^2}\,dx =$ $\frac{22\sqrt{2}-8}{105}$

4. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^2+\cos x)\sin x\,dx =$ 0

5. The formula for the inverse of the function $f(x)=\sqrt{x-2}$ is $f^{-1}(x) =$ x^2+2

   The domain of f^{-1} is $x \geq 0$

6. Evaluate $\int \tan\theta\,d\theta =$ $\ln|\sec\theta|+C$

7. State the definition of the natural logarithmic function : $\ln x = \int_1^x \frac{1}{t}\,dt, x>0$

8. (是非題) If $f(x_1)\neq f(x_2)$ implies $x_1 \neq x_2$ then we say that function f is one-to-one.

   True or false : False

II. 計算、證明題. (60 分)

1. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}\, dx$.

2. Evaluate $\int_0^{\frac{1}{4}} \frac{1}{\sqrt{1-4x^2}}\, dx$.

3. Evaluate $\int \frac{(\ln x)^3}{x}\, dx$.

4. Use logarithmic differentiation to find the derivative of $y = \frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}$.

5. Let $f(x) = 2x - \cos x$.
   (a) Show that f is one-to-one. (b) Find $(f^{-1})'(-1)$.

6. Prove the identity $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$.

108 學年度第一學期理工電資學院微積分 (3 學分) 期末考答案 2020.01.08

題號	答案	來源
1	$\frac{\pi}{4}$	6.6 – 習題 64*
2	$\frac{\pi}{12}$	6.6 – 例題 7
3	$\frac{1}{4}(\ln x)^4 + C$	6.2* – 習題 71*
4	$\frac{x\sqrt[4]{x^2+1}}{(3x+2)^5}(\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2})$	6.2* – 例題 14
5	(a)略　$(b)\frac{1}{2}$	6.1 – 例題 7*
6	略	6.6 – 例題 6

* 為非勾選習題、類似題.