

科目名稱: 微積分(下)(3 學分)

考試時間: 3 月 25 日第二節

I. 填充題. (45 分)

1. Let $H(t) = e^t + t^e$. Then $H'(t) = \underline{e^t + et^{e-1}}$.

2. Find $\int 3^{\sin \theta} \cos \theta \, d\theta = \underline{\frac{3^{\sin \theta}}{\ln 3} + C}$.

3. Find $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = \underline{1}$.

4. Find $\lim_{x \rightarrow 0^+} \sin x \ln x = \underline{0}$.

5. Find $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right) = \underline{\frac{1}{2}}$.

6. Let $f(x) = \ln(3 - \ln x)$.

Find the domain of $f = \underline{(0, e^3)}$ and its inverse function $f^{-1}(x) = \underline{e^{(3-x)}}$.

7. Evaluate $\int_1^e \frac{\log_8 x}{x} \, dx = \underline{\frac{1}{2 \ln 8}}$.

8. Find $\int_1^{e^2} \ln x \, dx = \underline{e^2 + 1}$.

II. 計算、證明題. (60 分)

1. Evaluate $\int e^x \sqrt{1 + e^x} \, dx$.

2. Differentiate $y = x^{\sqrt{x}}$.

3. Calculate $\int_0^1 \tan^{-1} x \, dx$.

4. Evaluate the integral $\int_0^1 \left(x^\pi + \pi^x + e^{\sin \frac{\pi x}{2}} \cos \frac{\pi x}{2} \right) \, dx$.

5. For what values of a , b and c is the following equation true ?

$$\lim_{x \rightarrow 0} \left(\frac{e^{2x}}{x^2} + a + \frac{b}{x} + \frac{c}{x^2} \right) = 0$$

6. Evaluate the integral $\int_{\sqrt{\pi}}^{\sqrt{\frac{\pi}{2}}} \theta^3 \cos(\theta^2) \, d\theta$.

題號	答案	來源
1	$\frac{2}{3}(1+e^x)^{\frac{3}{2}}+C$	6.3* - 習題 87
2	$x^{\sqrt{x}} \left[\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}} \right]$	6.4* - 例題 4
3	$\frac{\pi}{4} - \frac{1}{2} \ln 2$	7.1 - 例題 5
4	$\frac{1}{\pi+1} + \frac{\pi-1}{\ln \pi} + \frac{2}{\pi}(e-1)$	6.3*, 6.4*, 7.1 - 應用
5	$a = -2, b = -2, c = -1$	6.8 - 習題 100*
6	$\frac{1}{2} \left(\frac{\pi}{2} + 1 \right)$	7.1 - 習題 39*

* 為非勾選習題、類似題。