科目名稱: 微積分 (下)(3 學分)
考試時間: 5 月 6 日第二節

I. 填充題. (45 分)

1. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^5\theta\, d\theta = \underline{\dfrac{1}{120}}$

2. Evaluate $\displaystyle\int_0^{\pi} \sin^2 x\, dx = \underline{\dfrac{\pi}{2}}$

3. Find the limit of the function $f(x,y) = \dfrac{xy}{x^2+y^2}$ when $(x,y) \to (0,0)$ along the line $y = 2x$.

   Ans: $\underline{\dfrac{2}{5}}$

4. $f(x,y) = \begin{cases} x^2 + xy - y^3 & \text{, if } (x,y) \neq (2,1) \\ a & \text{, if } (x,y) = (2,1) \end{cases}$　is continuous on R^2, then $a = \underline{5}$

5. Evaluate $\displaystyle\int_0^1 \dfrac{1}{(1+x^2)^{\frac{3}{2}}}\, dx = \underline{\dfrac{\sqrt{2}}{2}}$

6. Evaluate $\displaystyle\int \dfrac{1}{x^2-4}\, dx = \underline{\dfrac{1}{4}\ln\left|\dfrac{x-2}{x+2}\right|} + C.$

7. Evaluate $\displaystyle\int \tan x \sec^3 x\, dx = \underline{\dfrac{1}{3}\sec^3 x} + C.$

8. Evaluate $\displaystyle\int_0^{\frac{1}{2}} x\sqrt{1-4x^2}\, dx = \underline{\dfrac{1}{12}}$

9. Evaluate $\displaystyle\int_0^{\infty} x e^{-x^2}\, dx = \underline{\dfrac{1}{2}}$

II. 計算、證明題. (60 分)

1. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2+y^4}$ does not exist.

2. (i) Evaluate $\displaystyle\int \sec^3 x\, dx.$

   (ii) Use the result of (i) to evaluate $\displaystyle\int \sqrt{1+x^2}\, dx.$

3. Evaluate $\displaystyle\int \dfrac{x}{\sqrt{3-2x-x^2}}\, dx.$

4. Evaluate $\displaystyle\int \dfrac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1}\, dx.$

5. Evaluate $\displaystyle\int_0^3 \frac{1}{x^2 - 6x + 5}\, dx.$

6. (i) Write out the form of the partial fraction decomposition of the function $f(x) = \dfrac{1}{(x^2 - 1)^2}.$

   (ii) Evaluate $\displaystyle\int \frac{1}{(x^2 - 1)^2}\, dx.$

108 學年度第二學期理工電資學院微積分 (3 學分) 第二次會考答案 2020.5.6

題號	答案	來源
1	略	14.2 − 例題 3
2	(i) $\dfrac{1}{2}\sec x \tan x + \dfrac{1}{2}\ln|\sec x + \tan x| + C$ (ii) $\dfrac{x}{2}\sqrt{1+x^2} + \dfrac{1}{2}\ln|\sqrt{1+x^2} + x| + C$	7.2 − 例題 8
3	$-\sqrt{3-2x-x^2} - \sin^{-1}\left(\dfrac{x+1}{2}\right) + C$	7.3 − 例題 7
4	$\dfrac{x^2}{2} + x + \ln|x-1| - \dfrac{2}{x-1} - \ln|x+1| + k$	7.4 − 例題 4
5	∞	7.8 − 習題 33
6	(i) $f(x) = \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2}\dfrac{C}{x-1} + \dfrac{D}{(x-1)^2}$ (ii) $\dfrac{1}{4}\ln|x+1| - \dfrac{1}{4}\left|\dfrac{1}{x+1}\right| - \dfrac{1}{4}\ln|x-1| - \dfrac{1}{4(x-1)} + C$	7.4 − 習題 21

* 為非勾選習題、類似題.