中原大學 108 學年度 □上學期 ■下學期 考試命題紙■ 第三次會考

科目名稱: 微積分 (下)(3 學分)
考試時間: 5 月 27 日第二節

I. 填充題. (45 分)

1. If $F(x,y)=\int_y^x \cos(e^t)\,dt$, then $F_x(x,y)=$ $\underline{\cos(e^x)}$, and $F_y(x,y)=$ $\underline{-\cos(e^y)}$.

2. If $g(r,s,t)=e^r\sin(st)$, then $g_{rst}=$ $\underline{e^r[\cos(st)-st\sin(st)]}$.

3. If $z=\ln(3x+2y), x=s\sin t, y=t\cos s$, then $\frac{\partial z}{\partial s}=$ $\underline{\frac{3\sin t-2t\sin s}{3x+2y}}$.

4. If $f(x,y,z)=x^2y+y^2z$, then the gradient of f is $\underline{<2xy, x^{+}2yz, y^2>}$, and the directional derivative of f at $(1,2,3)$ in the direction of $\vec{v}=<2,-1,2>$ is $\underline{1}$.

5. Let $f(x,y)=y^3+3x^2y-6x^2-6y^2+2.$ Then the local maximum value is $\underline{2}$, the local minimum value is $\underline{-30}$, and the saddle points are $\underline{(\pm 2,2)}$.

II. 計算、證明題. (60 分)

1. Find all the second partial derivatives of $f(x,y)=x^4y-2x^3y^2$.

2. If $g(s,t)=f(s^2-t^2,t^2-s^2)$ and f is differentiable, show that g satisfies the equation $t\frac{\partial g}{\partial s}+s\frac{\partial g}{\partial t}=0.$

3. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $x^3+y^3+z^3+6xyz=1.$

4. (a) If $f(x,y)=xe^y$, find the rate of change of f at the point $P(2,0)$ in the direction from P to $Q(\frac{1}{2},2)$.
   (b) In what direction does f have the maximum rate of change ? What is this maximum rate of change ?

5. Find the equations of the tangent plane and normal line at the point $(-2,1,-3)$ to the ellipsoid $\frac{x^2}{4}+y^2+\frac{z^2}{9}=3.$

6. Find the absolute maximum and minimum values of the function $f(x,y)=x^2-2xy+2y$ on the rectangle $D=\{(x,y)|\,0\le x\le 3, 0\le y\le 2\}$.

108 學年度第二學期理工電資學院微積分 (3 學分) 第三次會考答案 2020.5.27

題號	答案	來源
1	$f_{xx}=12x^2y-12xy^2, f_{xy}=4x^3-12x^2y, f_{yx}=4x^3-12x^2y, f_{yy}=-4x^3$	14.3 − 習題 53
2	略	14.5 − 例題 6
3	$\dfrac{\partial z}{\partial x}=\dfrac{-(x^2+2yz)}{z^2+2xy}, \dfrac{\partial z}{\partial y}=\dfrac{-(y^2+2xz)}{z^2+2xy}$	14.5 − 例題 9
4	$(a)1,(b)$The maximum rate of change is$\sqrt{5}$	14.6 − 例題 6
5	The tangent plane is $-(x+2)+2(y-1)-\frac{2}{3}(z+3)=0,$ The normal line is $\dfrac{x+2}{-1}=\dfrac{y-1}{2}=\dfrac{3(z+3)}{-2}$	14.6 − 例題 8
6	The absolute maximum is $9,$ the absolute minimum values is0	14.7 − 例題 7

* 為非勾選習題、類似題.