中原大學 108 學年度 □上學期 ■下學期 考試命題紙■第四次會考

科目名稱: 微積分 (下)(3 學分)
考試時間: 7 月 1 日第二節

I. 填充題. (45 分)

1. $\int_0^3 \int_1^2 x^2 y \, dydx =$ $\frac{27}{2}$

2. $\int_0^2 \int_0^{\frac{\pi}{2}} x \sin(y) \, dydx =$ 2

3. Find the volume of the solid bounded by the coordinate planes and the plane $3x + 2y + z = 6$.

   Ans: 6

4. Assume that $\iint_D f(x,y) \, dA = \int_{-2}^4 \int_a^b f(x,y) \, dxdy$, where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$. Then $a =$ $\frac{y^2 - 6}{2}$

5. $\iint_{\frac{1}{4} \le x^2 + y^2 \le 1} 1 \, dA =$ $\frac{3}{4}\pi$

6. $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2} \, dydx =$ $\frac{\pi}{4}(1 - e^{-4})$

7. Find the $6th$ Taylor polynomial $T_6(x)$ for $\sin(x)$ at $a = \frac{\pi}{2}$.
   $T_6(x) =$ $1 - \frac{1}{2}(x - \frac{\pi}{2})^2 + \frac{1}{24}(x - \frac{\pi}{2})^4 - \frac{1}{720}(x - \frac{\pi}{2})^6$

8. Find the $6th$ Taylor polynomial $T_6(x)$ for $\ln(1+x)$ at $a = 0$. $T_6(x) =$ $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6}$

9. Find the $3th$ Taylor polynomial $T_3(x)$ for $\frac{1}{\sqrt{1+x^2}}$ at $a = 0$. $T_3(x) =$ $1 - \frac{x^2}{2}$

II. 計算、證明題. (60 分)

1. Use the Midpoint Rule with $m = n = 2$ to estimate the value of the integral $\iint_R 3x - 2y \, dA$, where $R = \{(x,y) | 0 \le x \le 4, 1 \le y \le 5\}$.

2. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy-plane bounded by the line $y = 2x$ and $y = x^2$.

3. Evaluate $\iint_R 3x+4y\,dA$, where R is the region in the upper-half plane bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$.

4. Use Polar coordinates to find the volume of the solid below $z=\sqrt{x^2+y^2}$ and above the ring $1\le x^2+y^2\le 4$.

5. For the double integral $\iint_D e^{-y^2}\,dA$, where $D=\{(x,y)|0\le y\le 1, 0\le x\le y\}$.

   (a) Sketch the region of D. (b) Evaluate the value of the double integral.

6. Show that $\int_0^1 e^{-x^2}\,dx = 1+\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)n!}$.

108 學年度第二學期理工電資學院微積分 (3 學分) 第四次會考答案 2020.7.1

題號	答案	來源
1	0	15.1 – 例題 3*
2	$\frac{216}{35}$	15.2 – 例題 2
3	$\frac{56}{3}$	15.3 – 例題 1
4	$\frac{14\pi}{3}$	15.3 – 習題 20
5	(a)略　$(b)\frac{e^{-1}-1}{-2}$	15.2 – 習題 9*
6	略	講義

* 為非勾選習題、類似題.