I. 填充題. (45 分)

1. Evaluate $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9} = \dfrac{1}{a}$, where $a = \underline{\ 6\ }$

2. Let $f(x) = \dfrac{|x - 1|}{x - 1}$. Then $\lim\limits_{x \to 1^-} f(x) = \underline{\ -1\ }$ and $\lim\limits_{x \to 1^+} f(x) = \underline{\ 1\ }$

3. Define $f(2)$ so that the function $f(x) = \dfrac{x^2 + 2x - 8}{2 - x}$ is continuous at $x = 2$. $f(2) = \underline{\ -6\ }$

4. The equation for the tangent line to the curve $y = \dfrac{\sqrt{x}}{1 + x^2}$ which passes through the point $\left(1, \dfrac{1}{2}\right)$ is $y = mx + b$ where $m = \underline{\ \dfrac{-1}{4}\ }$ and $b = \underline{\ \dfrac{3}{4}\ }$

5. Let $f(x) = \begin{cases} x^2 & , \text{if } x \leq 2 \\ mx + b & , \text{if } x > 2 \end{cases}$. Find the values of m and b that make f differentiable everywhere. Then $m = \underline{\ 4\ }$ and $b = \underline{\ -4\ }$

6. Suppose that the targent line to the graph of a function f at $x = 1$ passes through the point $(4, 9)$ and that $f(1) = 3$. Then $f'(1) = \underline{\ 2\ }$

II. 計算、證明題. (60 分)

1. Show that $\lim\limits_{x \to 0} x^2 \sin \dfrac{1}{x} = 0$.

2. Evaluate $\lim\limits_{t \to 0} \left(\dfrac{1}{t\sqrt{1 + t}} - \dfrac{1}{t} \right)$.

3. Show that the function $f(x) = 1 - \sqrt{1 - x^2}$ is continuous on the interval $[-1, 1]$.

4. Show that there is a root of the equation $x^4 + x - 3 = 0$ between 1 and 2.

5. Let $f(x) = \sqrt{x}$. Use the definition of derivative to find the derivative of f.

6. Find an equation of the normal line to the curve $y = x + \sqrt{x}$ at the point $(1, 2)$.

109 學年度第一學期理工電資學院微積分 (3 學分) 第一次會考答案 2020.10.14

題號	答案	來源
1	略	1.6 − 例題 11*
2	$\dfrac{-1}{2}$	1.6 − 習題 29
3	略	1.8 − 例題 4
4	略	1.8 − 習題 53
5	$\dfrac{1}{2\sqrt{x}}$	2.2 − 例題 3
6	The normal line is $y - 2 = -\dfrac{2}{3}(x-1)$	2.3 − 習題 55

* 為非勾選習題、類似題.

* 證明題過程略過.