中原大學 109 學年度 ■上學期 □下學期 考試命題紙 ■期中考

科目名稱: 微積分 (上)(3 學分)
考試時間: 11 月 11 日第二節

I. 填充題. (45 分)

1. The $30th$ derivative of $\cos x$ is $-\cos x$

2. Let $f(x)=\sqrt{x+\sqrt{x+\sqrt{x}}}$, then $f'(x)=\dfrac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\dfrac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\dfrac{1}{2\sqrt{x}}\right)\right]$

3. Let $f(x)=\sin(\cos(\tan x))$, then $f'(x)=$ $\cos(\cos(\tan x))[-\sin(\tan x)]\sec^2 x$

4. Let f be differentiable and $F(x)=f(2f(xf(x)))$ with $f(0)=0$ and $f'(0)=3$,

   then $F'(0)=$ 0

5. Let $\sin(x+y)=y^3\cos x$, then $y'=$ $\dfrac{y^3\sin x+\cos(x+y)}{3y^2\cos x-\cos(x+y)}$

6. The best estimation of $\tan 2^\circ$ by a linear approximation is the value $=\dfrac{a}{90}$, where $a=$ π

7. The critical numbers(points) of $f(x)=x^{\frac{3}{5}}(4-x)$ are 0 and $\dfrac{3}{2}$

8. The absolute maximum values of $f(x)=x^3-x^2$, $-\dfrac{1}{2}\le x\le 4$ is 48

II. 計算、證明題. (60 分)

1. Differentiate $f(x)=\dfrac{\sec x}{1+\tan x}$. For what values of x does the graph of f have a horizontal tangent ?

2. Find (a) $\lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta+\tan\theta}$ (b) $\lim\limits_{x\to 0}\dfrac{\cos x-1}{2x^2}$

3. (a) Find y' if $x^3+y^3=6xy$.
   (b) Find the tangent to the folium of Descartes $x^3+y^3=6xy$ at the point $(3,3)$.
   (c) At what point in the first quadrant is the tangent line horizontal ?

4. Show that $y''=-48\dfrac{x^2}{y^7}$ if $x^4+y^4=16$

5. Use a linear approximation(or differentials) to estimate $\sqrt[3]{1001}$.

6. If a and b are positive numbers, find the maximum value of $f(x)=x^a(1-x)^b, 0\le x\le 1$.

109 學年度第一學期理工電資學院微積分 (3 學分) 期中考答案 2020.11.11

題號	答案	來源
1	$x = n\pi + \frac{\pi}{4}, n \in \mathbb{Z}$	2.4 – 例題 2
2	$(a)\ \frac{1}{2}\quad (b)\ \frac{-1}{4}$	2.4 – 習題 45、47
3	$(a)\ y' = \frac{2y - x^2}{y^2 - 2x}\quad (b)\ y - 3 = -(x - 3)\quad (c)\ (x, y) = (2^{\frac{4}{3}}, 2^{\frac{5}{3}})$	2.6 – 例題 2
4	略	2.6 – 例題 4
5	$10 + \frac{1}{300}$	2.9 – 習題 25
6	The maximum value is $\left(\frac{a}{a+b}\right)^a \left(\frac{b}{a+b}\right)^b$	3.1 – 習題 57

* 為非勾選習題、類似題.
* 證明題過程略過.