科目名稱: 微積分 (上)(3 學分)
考試時間: 12 月 9 日第二節

I. 填充題. (45 分)

1. The local maximum value of the function $g(x) = x + 2\sin x$ in the interval $[0, 2\pi]$ is $\dfrac{2\pi}{3} + \sqrt{3}$

2. Suppose that $f(0) = 4$ and $f'(x) \leq 3$ for all values of x. How large can $f(2)$ possibly be ?

   Ans： 10

3. If it is known that $\displaystyle\int_1^9 f(x)\,dx = 13$ and $\displaystyle\int_6^9 f(x)\,dx = 4$, find $\displaystyle\int_1^6 f(x)\,dx = $ 9

4. Express $\displaystyle\lim_{n\to\infty} \sum_{i=1}^n (x_i^3 + x_i \sin x_i)\Delta x$ as an integral on the interval $[0, \pi]$. Ans： $\displaystyle\int_0^\pi (x^3 + x\sin x)\,dx$

5. Let $f(x) = x\sqrt{6-x}$. The increasing interval of f is $(-\infty, 4)$. The decreasing interval of

   f is $(4, 6)$

6. Find $\displaystyle\lim_{x\to\infty} (\sqrt{9x^2 + x} - 3x) = $ $\dfrac{1}{6}$

7. Find $\displaystyle\lim_{x\to\infty} (x - \sqrt{x}) = $ ∞

8. Let $f(x) = \dfrac{x^3}{(x+1)^2}$. The slant asymptote of f is $y = x - 2$

II. 計算、證明題. (60 分)

1. Show that the equation $2x + \cos x = 0$ has exactly one real root.

2. Use the Mean Value Theorem to prove the inequality $|\sin a - \sin b| \leq |a - b|$, for all a and b.

3. Use the definition of the integral $\displaystyle\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^n f(x_i)\Delta x$ where $\Delta x = \dfrac{b-a}{n}$,

   $x_i = a + i \cdot \Delta x$ to prove that $\displaystyle\int_a^b x\,dx = \dfrac{b^2 - a^2}{2}$.

4. Evaluate the integral by interpreting it in terms of areas $\displaystyle\int_{-3}^0 (1 + \sqrt{9 - x^2})\,dx$.

5. Let $f(x) = \dfrac{x}{x-1}$.

   (a) Find the intervals on which f is increasing or decreasing.

   (b) Find the intervals on which f is concave upward or downward.

   (c) Sketch the graph of f.

6. Find the horizontal and vertical asymptotes of the function $f(x) = \dfrac{\sqrt{2x^2+1}}{3x-5}$.

109 學年度第一學期理工電資學院微積分 (3 學分) 第二次會考答案 2020.12.9

題號	答案	來源
1	略	3.2 − 習題 19
2	略	3.2 − 習題 31
3	略	3.4 − 習題 32*
4	$3 + \dfrac{9}{4}\pi$	4.2 − 習題 27
5	$(a)\, f(x)$ is decreasing on $(-\infty, 1), (1, \infty)$. $(b)\, f(x)$ is concave upword on $(1, \infty)$, is concave downword on $(-\infty, 1)$. (c)略	3.5 − 習題 9
6	The vertical asymptotes is $x = \dfrac{5}{3}$, the horizontal asymptotes is $y = \dfrac{\pm\sqrt{2}}{3}$	3.4 − 例題 4

* 為非勾選習題、類似題.
證明、作圖題過程略過.