I. 填充題. (45 分)

1. If $F(x) = \int_x^0 \sqrt{1 + \sec t}\, dt$, then $F'(x) = \underline{\ -\sqrt{1 + \sec x}\ }$

2. If $f(1) = 12$, f' is continuous and $\int_1^4 f'(x)\, dx = 16$, then $f(4) = \underline{\ 28\ }$

3. Evaluate $\int_{-1}^1 t(1-t)^2\, dt = \underline{\ -\dfrac{4}{3}\ }$

4. Evaluate $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} (x^3 + x^4 \tan x)\, dx = \underline{\ 0\ }$

5. If $f(x) = \int_2^x \sqrt{1 + t^2}\, dt$, then $(f^{-1})'(0) = \underline{\ \dfrac{1}{\sqrt{5}}\ }$

6. Evaluate $\int_e^4 \dfrac{1}{x \ln x}\, dx = \underline{\ \ln(\ln 4)\ }$

7. Evaluate $\tan(\sin^{-1}(\dfrac{2}{3})) = \underline{\ \dfrac{2}{\sqrt{5}}\ }$

8. Evaluate $\int \tan x\, dx = \underline{\ -\ln|cos x| + C\ }$

9. Let $f(x) = \sqrt{x^3 + 4x + 4}$. Find $(f^{-1})'(3) = \underline{\ \dfrac{6}{7}\ }$

II. 計算、證明題. (60 分)

1. Evaluate the limit by first recognizing the sum as a Riemann sum：$\lim\limits_{n \to \infty} \dfrac{1}{n} \left(\sqrt{\dfrac{1}{n}} + \sqrt{\dfrac{2}{n}} + ... + \sqrt{\dfrac{n}{n}} \right).$

2. Evaluate (a) $\displaystyle\int \dfrac{x}{\sqrt{x^2 + 1}}\, dx.$ (b) $\displaystyle\int x^3 \sqrt{x^2 + 1}\, dx.$

3. Let $f(x) = \sqrt{x - 2}$. (a) Find $f^{-1}(x)$. (b) Sketch the graphs of f and f^{-1} on the same axes. (c) Find $f^{-1}(2)$. (d) Find $(f^{-1})'(2)$.

4. Use logarithmic differentiation to find the derivative of $y = \dfrac{(x^3 + 1)^4 \sin^2 x}{x^{\frac{1}{3}}}, \ 0 < x < \pi.$

5. Find (a) $\displaystyle\int_0^{\frac{1}{4}} \dfrac{1}{\sqrt{1 - 4x^2}}\, dx.$ (b) $\displaystyle\int \dfrac{x}{x^4 + 9}\, dx.$

6. Differentiate (a) $y = \tan^{-1} \sqrt{x}.$ (b) $y = \sqrt{x^2 - 1}\ \sec^{-1} x.$

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題號	答案	來源
1	$\dfrac{2}{3}$	4.3 − 習題 68
2	$(a)\sqrt{x^2+1}+C.$ $(b)\dfrac{1}{5}(x^2+1)^{\frac{5}{2}}-\dfrac{1}{3}(x^2+1)^{\frac{3}{2}}+C$	4.5 − (a) 例題 5*, (b) 習題 30
3	$(a)f^{-1}(x)=x^2+2.$ (b)略. $(c)f^{-1}(2)=6.$ $(d)(f^{-1})'(2)=4$	6.1 − 習題 36
4	$y'=\dfrac{(x^3+1)^4\sin^2 x}{x^{\frac{1}{3}}}\left(\dfrac{12x^2}{x^3+1}+\dfrac{2\cos x}{\sin x}-\dfrac{1}{3x}\right)$	6.2* − 習題 64
5	$(a)\dfrac{\pi}{12}.$ $(b)\dfrac{1}{6}\tan^{-1}\left(\dfrac{x^2}{3}\right)+C$	6.6 − (a) 例題 7, (b) 例題 9
6	$(a)\dfrac{1}{2\sqrt{x}(1+x)}.$ $(b)\dfrac{x}{\sqrt{x^2-1}}\sec^{-1}x+\sqrt{x^2-1}\dfrac{1}{x\sqrt{x^2-1}}$	6.6 − (a) 習題 23, (b) 習題 30

* 為非勾選習題、類似題.
證明、作圖題過程略過.