

科目名稱: 微積分(下)(3學分)

考試時間: 3月24日第二節

I. 填充題. (45分)

1. The domain of the function $f(x) = \sqrt{3 - e^{2x}}$ is $(-\infty, \underline{\frac{\ln 3}{2}}]$

2. $\lim_{x \rightarrow 0} (1 - 2x)^{\frac{1}{x}} = \underline{e^{-2}}$

3. If $\int_0^{\frac{\pi}{6}} \sqrt{1 + \cos 2x} dx = \frac{a}{2}$, then $a = \underline{\sqrt{2}}$

4. The critical number of the function $f(x) = xe^{-x}$ is 1

5. Evaluate the integral: $\int_0^4 2^s ds = \underline{\frac{15}{\ln 2}}$

6. If f' is continuous, $f(2) = 0$, and $f'(2) = 7$, then $\lim_{x \rightarrow 0} \frac{f(2+3x) + f(2+5x)}{x} = \underline{56}$

7. $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x} = \underline{1}$

8. Evaluate the integral: $\int_1^e \ln x dx = \underline{1}$

9. If $\int t^2 e^t dt = p(t)e^t + C$, then $p(t) = \underline{t^2 - 2t + 2}$

II. 計算、證明題. (60分)

1. For what values of a and b is the following equation true ?

$$\lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x^3} + a + \frac{b}{x^2} \right) = 0$$

2. Evaluate the integral: $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2) d\theta$.

[Hint: Use the substitution rule to simplify the above integral first.]

3. Find the average value of the function $f(x) = \sin^2 x \cos^3 x$ on the interval $\left[0, \frac{\pi}{2}\right]$.

4. Find the y' . (a) $y = x^x$ (b) $x^y = y^x$.

5. Find (a) $\lim_{x \rightarrow 0^+} x^x$ (b) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$.

6. Find (a) $\int \tan^2 \theta \sec^4 \theta d\theta$ (b) $\int \tan^5 x \sec^3 x dx$.

109 學年度第二學期理工電資學院微積分 (3 學分) 第一次會考答案 2021.3.24

題號	答案	來源
1	$a = \frac{2}{3}, b = -1$	6.8 - 習題 100*
2	$-\frac{\pi}{4} - \frac{1}{2}$	7.1 - 習題 39
3	$\frac{4}{15\pi}$	7.2 - 習題 55*
4	(a) $y' = x^x(1 + \ln x)$, (b) $y' = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}$	6.4* - 習題 35,54
5	(a) 1, (b) $\frac{1}{2}$	6.8 - 例題 10, 習題 53
6	(a) $\frac{1}{3} \tan^3 \theta + \frac{1}{5} \tan^5 \theta + C$, (b) $\frac{1}{3} \sec^3 x - \frac{2}{5} \sec^5 x + \frac{1}{7} \sec^7 x + C$	7.2 - 習題 22,28

* 為非勾選習題、類似題.

* 證明題過程略過.