中原大學 109 學年度 □上學期 ■下學期 考試命題紙 ■期中考

科目名稱: 微積分 (下)(3 學分)
考試時間: 4 月 21 日第二節

I. 填充題. (45 分)

1. $\int_0^{\frac{1}{2}} \frac{\arctan(2x)}{1+4x^2}\,dx = \frac{\pi^2}{a}$, where $a =$ $\underline{\quad 64 \quad}$

2. $\int_0^{\frac{3}{4}} \frac{1}{\sqrt{9-4x^2}}\,dx =$ $\underline{\quad \frac{\pi}{12} \quad}$

3. $\int \frac{1}{\sqrt{3-2x-x^2}}\,dx = A \arcsin\left(\frac{1}{2}f(x)\right) + C$, where $A =$ $\underline{\quad 1 \quad}$, $f(x) =$ $\underline{\quad 1+x \quad}$
and C is an arbitrary constant.

4. $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\,dx = \ln\left|\frac{x-1}{x+1}\right| + f(x) + C$, where $f(x) =$ $\underline{\quad \frac{x^2}{2} + x - \frac{2}{x-1} \quad}$
and C is an arbitrary constant.

5. Evaluate the improper integral $\int_2^5 \frac{1}{\sqrt{x-2}}\,dx$ if possible. Ans: $\underline{\quad 2\sqrt{3} \quad}$

6. Let $f(x,y) = \frac{x^2y}{x^3+y^3}$ for all $(x,y) \neq (0,0)$.
   (a) $\lim_{x\to 0} f(x,0) =$ $\underline{\quad 0 \quad}$
   (b) $\lim_{x\to 0} f(x,x) =$ $\underline{\quad \frac{1}{2} \quad}$
   (c) Together what do (a) and (b) tell us about $\lim_{(x,y)\to(0,0)} f(x,y)$? Ans: $\underline{\quad \text{does not exist} \quad}$
   (Whether this limit exists or not ?)

II. 計算、證明題. (60 分)

1. Evaluate $\int \frac{\sqrt{x+4}}{x}\,dx$.

2. Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$ if possible.

3. Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$ if possible.

4. If $f(x,y) = \frac{xy^2}{x^2+y^4}$, does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Why?

5. Find $\lim_{(x,y)\to(0,0)} \dfrac{3x^2y}{x^2+y^2}$ if it exists.

6. Evaluate $\displaystyle\int \frac{4x^2-3x+2}{4x^2-4x+3}\,dx.$

109 學年度第二學期理工電資學院微積分 (3 學分) 期中考答案 2021.4.21

題號	答案	來源
1	$2\sqrt{x+4}+2\ln\lvert\sqrt{x+4}-2\rvert-2\ln\lvert\sqrt{x+4}+2\rvert+C$	7.4 – 例題 9
2	π	7.8 – 例題 3
3	$\dfrac{\pi}{2}$	7.8 – 習題 32
4	略	14.2 – 例題 3
5	0	14.2 – 例題 4
6	$x+\dfrac{1}{8}\ln(4x^2-4x+3)-\dfrac{\sqrt{2}}{8}\tan^{-1}\left(\dfrac{2x-1}{\sqrt{2}}\right)+C$	7.4 – 例題 6

* 為非勾選習題、類似題.
* 證明題過程略過.