

科目名稱: 微積分(上)(3學分)  
 考試時間: 10月13日第二節

I. 計算、證明題. (60分)

1. Find the limits in the following: (1)  $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$  (2)  $\lim_{x \rightarrow 0} x \sin \left( \frac{1}{x} \right)$ .

2. Find the derivative of the function  $f(x) = \frac{1}{x^2}$  by the limit process.

3. Determine whether the function  $f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ 4x - 3, & x > 2 \end{cases}$  is differentiable at  $x = 2$ .

4. Let  $f(x) = \frac{x+2}{x^2-3x-10}$ . Find the  $x$ -values at which  $f$  is not continuous.

Which of the discontinuities are removable?

5. Use the Intermediate Value Theorem to show that  $f(x) = x^2 - 2 - \cos x$  has at least one zero in the interval  $[0, \pi]$ .

6. Determine all vertical asymptotes of the graph of the function  $h(x) = \frac{x^2 + 2x - 8}{x^2 - 4}$ .

II. 填充題. (45分)

1. Evaluate  $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x-4} = \underline{\frac{1}{6}}$

2. Evaluate  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{2x^2} = \underline{-\frac{1}{4}}$

3. If  $f(x) = \begin{cases} 3x^2, & x \geq 1 \\ ax - 4, & x < 1 \end{cases}$  is continuous on the entire real number line, then  $a = \underline{7}$

4. Evaluate  $\lim_{x \rightarrow 0^-} [x] = \underline{-1}$ , where  $[x]$  is the greatest integer function.

5. Evaluate  $\lim_{x \rightarrow \pi^+} \frac{\sqrt{x}}{\csc x} = \underline{0}$

6. Is the following statement true or false? "The graph of the function  $f(x) = x^{\frac{1}{3}}$  has a vertical tangent line at  $x = 0$ . So,  $f$  is not differentiable at  $x = 0$ ." Ans : True (True or False)

7. Let  $f(x) = 4 - x^2$  and  $g(x) = \sqrt{x+1}$ . Find  $\lim_{x \rightarrow 1} g(f(x)) = \underline{2}$

8. Let  $\lim_{x \rightarrow c} f(x) = \infty$  and  $\lim_{x \rightarrow c} g(x) = -2$ . Find  $\lim_{x \rightarrow c} \frac{g(x)}{f(x)} = \underline{0}$

9. The tangent line to the function  $y = g(x)$  at the point  $(4, 5)$  passes through the point  $(7, 0)$ .

Find  $g'(4) = \underline{-\frac{5}{3}}$

## 110 學年度第一學期理工電資學院微積分 (3 學分) 第一次會考答案 2021.10.13

題號	答案	來源
1	(1) $\frac{2}{3}$ , (2) 0	1.3 – 習題 74.95
2	$\frac{-2}{x^3}$	1.3 – 習題 90*
3	略	1.4 – 習題 23*
4	$f$ is discontinuous at $-2$ and $5$ , $f$ is a removable discontinuity at $-2$	1.4 – 習題 47
5	略	1.4 – 習題 85
6	The vertical asymptote is $x = -2$	1.5 – 例題 3

\* 為非勾選習題、類似題。  
證明題過程略過。