

科目名稱: 微積分(上)(3學分)

考試時間: 11 月 10 日第二節

I. 計算、證明題. (60 分)

- Find the derivative of  $y = \sin \sqrt{\cot(3\pi x)}$ .
- Given  $xy - 1 = 2x + y^2$ , find  $\frac{d^2y}{dx^2}$ .
- Find the absolute extrema of  $f(x) = 2 \sin x - \cos(2x)$  on the interval  $[0, 2\pi]$ .
- Use Rolle's Theorem to show that the function  $f(x) = x^5 + x^3 + x + 1$  can not have two zeros in the interval  $[-1, 0]$ .
- Let  $0 < a < b$ . Use the Mean Value Theorem to show that  $\sqrt{b} - \sqrt{a} < \frac{b-a}{2\sqrt{a}}$ .
- Find the absolute extrema of  $f(x) = 2x - 3x^{\frac{2}{3}}$  on the interval  $[-1, 3]$ .

II. 填充題. (45 分)

- Let  $f(x) = \frac{2}{\sqrt[3]{x}} + 3 \cos x$ . Then  $f'(x) = \underline{-\frac{2}{3}x^{-\frac{4}{3}} - 3 \sin x}$
- Let  $f(x) = \frac{x^{\frac{3}{2}} - x}{3x - x^{\frac{1}{2}}}$ . Then  $f'(4) = \frac{9}{a}$ , where  $a = \underline{100}$
- Let  $f(x) = (3x - 2x^2)^{30}$ . Then  $f'(x) = \underline{30(3 - 4x)(3x - 2x^2)^{29}}$
- The equation of the tangent line to the lemniscate  $3(x^2 + y^2)^2 = 100xy$  at the point  $(3, 1)$  is  
 $y - 1 = m(x - 3)$ , where  $m = \underline{\frac{13}{9}}$
- The equation of the tangent line to  $y = \sqrt{x^2 + 8x}$  at the point  $(1, 3)$  is  $y = b + ax$ . Then  
 $2a + b = \underline{\frac{14}{3}}$
- Given  $x^2(x^2 + y^2) = y^2$ , find  $\frac{dy}{dx} = \underline{\frac{x(2x^2 + y^2)}{y(1 - x^2)}}$

7. The critical numbers of  $h(x) = \sin^2 x + \cos x$  in the range  $0 < x < 2\pi$  are  $x = \frac{\pi}{3}, \underline{\pi}$

and  $\underline{\frac{5\pi}{3}}$

8. Let  $f(x) = x^4 + x + 3$  for  $0 \leq x \leq 2$ . Find a point  $c$  whose existence is guaranteed by the

Mean Value Theorem. Answer:  $c = 3^p$ , where  $p = \underline{\frac{1}{3}}$

題號	答案	來源
1	$y' = \cos \sqrt{\cot(3\pi x)} \cdot \frac{1}{2\sqrt{\cot 3\pi x}} \cdot (-\csc^2(3\pi x)) \cdot (3\pi)$	2.4 - 習題 53
2	$\frac{10}{(x-2y)^3}$	2.5 - 習題 52*
3	The absolute max. is $f(\frac{\pi}{2}) = 3$ , absolute min. is $f(\frac{7}{6}\pi) = f(\frac{11\pi}{6}) = -\frac{3}{2}$	3.1 - 例題 4
4	略	3.2 - 習題 65
5	略	3.2 - 習題 85
6	The absolute max. is $f(0) = 0$ , absolute min. is $f(-1) = -5$	3.1 - 例題 3

\* 為非勾選習題、類似題.

\* 證明題過程略過.