科目名稱: 微積分 (上)(3 學分)
考試時間: 12 月 8 日第二節

I. 計算、證明題. (60 分)

1. Find (a) $\displaystyle\lim_{x \to -\infty} (x + \sqrt{x^2 + 3})$ (b) $\displaystyle\lim_{x \to -\infty} \frac{3x}{\sqrt{x^2 + 1}}$.

2. Use differentials to estimate $\sqrt{99.4}$.

3. Let $f(x) = \dfrac{x}{\sqrt{x^2 + 2}}$.

   (4%) (a) Find the horizontal asymptotes of $f(x)$.

   (2%) (b) Find the intervals on which f is increasing.

   (4%) (c) Determine the intervals on which the graph of f is concave upward or concave downward.
   $\left(\text{Hint: } f'(x) = \dfrac{2}{(x^2 + 2)^{\frac{3}{2}}}, f''(x) = \dfrac{-6x}{(x^2 + 2)^{\frac{5}{2}}}\right)$

4. Let $f(x) = x^4 - 4x^3$. Determine the points of inflection of f.

5. Find the relative extrema of $f(x) = \dfrac{x^4 + 1}{x^2}$.

6. Sketch the graph of $f(x) = \dfrac{2(x^2 - 9)}{x^2 - 4}$ by considering the increasing/decreasing intervals, concavity, asymptotes and symmetry. $\left(\text{Hint: } f'(x) = \dfrac{20x}{(x^2 - 4)^2}, f''(x) = \dfrac{-20(3x^2 + 4)}{(x^2 - 4)^3}\right)$

II. 填充題. (45 分)

1. Let $f(x) = x\sqrt{16 - x^2}$. The increasing interval of f is $\underline{(-2\sqrt{2}, 2\sqrt{2})}$

2. The relative maximum of the function $f(x) = \dfrac{x^2}{x^2 - 9}$ occurs at $x = \underline{0}$

3. The point of inflection of $g(x) = x^{\frac{1}{3}}$ occurs at $x = \underline{0}$

4. Let $f(x) = \dfrac{x^2 + 1}{x^2 - 4}$. Determine the open intervals on which the graph of $f(x)$ is concave

   downward. Ans: $\underline{(-2, 2)}$

5. Let $f(x) = \dfrac{\sin 4x}{x}$. The horizontal asymptote of f is $\underline{y = 0}$

6. Find $\lim\limits_{x \to \infty} \dfrac{x - \cos x}{x} = \underline{1}$

7. Let $f(x) = \dfrac{2x^2 - 4x}{x + 1}$. The slant asymptote of f is $\underline{y = 2x - 6}$

8. Let $y = \sqrt{x}$. Find the differential $dy = \underline{\dfrac{1}{2\sqrt{x}}\, dx}$

9. Let $y = x^2 \sec x$. Find the differential $dy = \underline{2x \sec x + x^2 \sec x \tan x\, dx}$

110 學年度第一學期理工電資學院微積分 (3 學分) 第二次會考答案 2021.12.8

題號	答案	來源
1	$(a)0, \quad (b) - 3$	$3.5 - $ 習題 $43, 25^*$
2	略	$3.9 - $ 習題 43
3	(a) Horizontal asymptotes of $f(x) : y = 1, -1$ (b) f is increasing on $(-\infty, \infty)$ (c) f is concave upward $(-\infty, 0),$ concave downward $(0, \infty)$	$3.6 - $ 例題 3
4	Point of inflection of $f : x = 0, 2$	$3.4 - $ 例題 3
5	The relative minimum is $f(\pm 1) = 2$	$3.3 - $ 例題 4
6	略	$3.6 - $ 例題 1

證明題、做圖題過程略過.

* 為非勾選習題、類似題.