I. 計算、證明題. (60 分)

1. Use logarithmic differentiation to find $\dfrac{dy}{dx}$, where $y = \dfrac{x^2\sqrt{3x-2}}{(x+1)^2}$ $\left(x > \dfrac{2}{3}\right)$.

2. Find (1) $\displaystyle\int_1^4 \dfrac{u-2}{\sqrt{u}}\, du$ (2) $\displaystyle\int_0^4 |x^2 - 9|\, dx$.

3. Find $\displaystyle\int \dfrac{x^2-1}{\sqrt{2x-1}}\, dx$.

4. Let $f(x) = \sqrt{x}$, $x_i = \dfrac{i^2}{n^2}$ $(0 \le i \le n)$, $\triangle x_i = x_i - x_{i-1}$ $(1 \le i \le n)$, and

   $||\triangle|| = \max\{\triangle x_1, \triangle x_2, ..., \triangle x_n\}$. Find (1) $\|\triangle\|$ (2) $\displaystyle\sum_{i=1}^n f(x_i)\triangle x_i$ (3) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^n f(x_i)\triangle x_i$

5. Find $\displaystyle\int \dfrac{x^4+x-4}{x^2+2}\, dx$.

6. Find $\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{1+\tan^2 x}\, dx$.

II. 填充題. (45 分)

1. Let c_i be any point in the ith subinterval of a partition $\triangle$ of the interval $[0, 3]$. Write the following limit as a definite integral. Then $\displaystyle \lim_{||\triangle|| \to 0} \sum_{i=1}^{n} \sqrt{c_i^2 + 4}\, \triangle x_i = $ _____ $\displaystyle \int_0^3 \sqrt{x^2 + 4}\, dx$ _____

2. If $\displaystyle \int_{-1}^{1} f(x)\, dx = 0$ and $\displaystyle \int_0^1 f(x)\, dx = 5$, then $\displaystyle \int_{-1}^{0} f(x)\, dx = $ ___-5___

3. Evaluate $\displaystyle \int_{-2}^{2} \sqrt{4 - x^2}\, dx = $ ___2π___

4. If the average value of $f(x) = \sin x$ over the interval $[0, \pi]$ is $\dfrac{k}{\pi}$, then $k = $ ___2___

5. If $\displaystyle F(x) = \int_0^{x^3} \sin t^2\, dt$ and $F'(x) = ax^b \sin x^c$, then $a + b + c = $ ___11___

6. Evaluate $\displaystyle \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^3 x \cos x + \sin x \cos x)\, dx = $ ___0___

7. If $f(x) = \ln |\cos x|$, then $f'(x) = $ ___$\dfrac{-\sin x}{\cos x}$___

8. If $\displaystyle \int_1^e \frac{(1 + \ln x)^2}{x}\, dx = \frac{m}{3}$, then $m = $ ___7___ (m is an integer)

9. Evaluate $\displaystyle \int \frac{\cos t}{1 + \sin t}\, dt = $ ___$\ln|1 + \sin t|$___ $+ C$

110 學年度第一學期理工電資學院微積分 (3 學分) 期末考答案 2022.1.12

題號	答案	來源
1	$y' = \dfrac{x^2\sqrt{3x-2}}{(x+1)^2}\left[\dfrac{2}{x} + \dfrac{3}{2(3x-2)} - \dfrac{2}{x+1}\right]$	5.1 — 習題 77
2	$(a)\ \dfrac{2}{3},\quad (b)\ \dfrac{64}{3}$	4.4 — 習題 17,27
3	$\dfrac{1}{20}(2x-1)^{\frac{5}{2}} + \dfrac{1}{6}(2x-1)^{\frac{3}{2}} - \dfrac{3}{4}(2x-1)^{\frac{1}{2}} + C$	4.5 — 習題 57
4	$(a)\ \dfrac{2n-1}{n^2},\ (b)\ \dfrac{2}{3} + \dfrac{1}{2n} - \dfrac{1}{6n^2},\ (c)\ \dfrac{2}{3}$	4.3 — 例題 1*
5	$\dfrac{x^3}{3} - 2x + \dfrac{1}{2}\ln(x^2+2) + C$	5.2 — 習題 21
6	$\ln(\sqrt{2}+1)$	5.2 — 例題 10

證明題、做圖題過程略過.

* 為非勾選習題、類似題.