

科目名稱: 微積分(下)(3 學分)

考試時間: 3 月 23 日第二節

I. 計算、證明題. (60 分)

1. Evaluate $\int \frac{x+2}{\sqrt{4-x^2}} dx$

2. Find (a) $\frac{d}{dx} [\arctan(e^x)]$ (b) $\frac{d}{dx} [\arcsin(\sqrt{x})]$.

3. Evaluate $\lim_{x \rightarrow 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$.

4. Find (a) $\frac{d}{dx} \log_5 \sqrt{x^2-1}$. (b) $\frac{d}{dx} [x^x]$.

5. Evaluate $\int_0^{\sqrt{2}} x e^{-\frac{x^2}{2}} dx$.

6. Let $f(x) = \frac{x+6}{x-2}, x > 2$. (a) Find the inverse function of f . (b) Find $(f^{-1})'(3)$.

II. 填充題. (45 分)

1. If $f(x) = x^3 + 3x - 1$, then $(f^{-1})'(-5) = \underline{\frac{1}{6}}$

2. Evaluate $\int_0^2 \frac{1}{x^2 - 2x + 2} dx = \underline{\frac{\pi}{2}}$

3. If $f(x) = e^{-x} \ln x$, then the slope of the tangent line at $(1, 0)$ is: $\underline{e^{-1}}$

4. Evaluate $\int_{-1}^2 2^x dx = \underline{\frac{7}{\ln 4}}$

5. Evaluate $\lim_{x \rightarrow \infty} x^{\frac{1}{x}} = \underline{1}$

6. Evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x = \underline{e}$

7. Evaluate $\sin \left(\arctan \frac{3}{4} \right) = \underline{\frac{3}{5}}$

8. If $\arcsin(3x - \pi) = \frac{1}{2}$, then $x = \underline{\frac{1}{3} \left[\sin\left(\frac{1}{2}\right) + \pi \right]}$

9. The area of the region bounded by the graph of $f(x) = \frac{1}{\sqrt{3x-x^2}}$, the x -axis, and the lines

$$x = \frac{3}{2} \text{ and } x = \frac{9}{4} \text{ is } \underline{\frac{\pi}{6}}$$

110 學年度第 2 學期理工電資學院微積分 (3 學分) 第一次會考答案 2022.3.23

題號	答案	來源
1	$-\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) + C$	5.8 – 例題 3
2	(a) $\frac{e^x}{1+e^{2x}}$, (b) $\frac{1}{2\sqrt{x-x^2}}$	5.7 – 例題 4-c, 習題 45
3	$\frac{1}{2}$	5.6 – 例題 7
4	(a) $\frac{x}{(\ln 5)(x^2-1)}$, (b) $x^x(\ln x + 1)$	5.5 – 習題 53, 例題 5-d
5	$-e^{-1} + 1$	5.4 – 習題 114
6	(a) $x = \frac{2y+6}{y-1}$ or $y = \frac{2x+6}{x-1}$, (b) -2	5.3 – 習題 69

* 為非勾選習題、類似題.

證明題過程略過.