科目名稱: 微積分 (上)(3 學分)
考試時間: 10 月 12 日第二節

I. 計算、證明題. (80 分)

1. Find the limits (a) $\lim\limits_{x \to 3} \dfrac{\sqrt{x+1}-2}{x-3}$　　(b) $\lim\limits_{x \to 1} \dfrac{x^2+3x-4}{x^2-3x+2}$

2. Without using L'Hospital's Rule,　(a) prove $\lim\limits_{x \to 0^+} \dfrac{\sin x}{x} = 1$　　(b) find $\lim\limits_{x \to 0} \dfrac{\sin 4x}{\sin 3x}$.

3. Describe the interval(s) on which the function $f(x)$ is continuous.

   (a) $f(x) = \dfrac{x}{x^2+x+2}$　　　　(b) $f(x) = \dfrac{x^2-1+\sin 3x}{x+1}$.

4. Use the Intermediate Value Theorem to show that the function $f(x) = x^3 - 2x^2 + 5$ has a zero in the interval $[-2, 0]$

5. Let $f(x) = \dfrac{x^2+2x-8}{(x^2-4)(x-1)}$. Find all vertical asymptotes of $f(x)$.

6. (a) Show that $f(x) = |x-2|$ is continuous at $x = 2$.
   (b) Show that $f(x) = |x-2|$ is not differentiable at $x = 2$.

7. Find the derivative of the function by the limit process. (a) $f(x) = 4x + 5$　　(b) $f(x) = \dfrac{1}{x}$

8. Let $f(x) = \begin{cases} x^2 \sin \dfrac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$.

   (a) Show that $f(x)$ is differentiable at $x = 0$.　　(b) Find $f'(0)$.

9. Find the derivative of the function.

   (a) $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2}$　　(b) $f(x) = x^2 + \sqrt{x} + 4\cos x + x^{\frac{1}{9}}$

10. Find an equation of the tangent line to the graph of $f(x) = \dfrac{2}{\sqrt[3]{x^4}}$ at $(1, 2)$.

II. 填充題. (25 分)

1. Evaluate $\lim\limits_{x \to 0} \dfrac{(1 - \cos x)^2}{x} = \underline{\ 0\ }$

2. If $g(x) = \begin{cases} \dfrac{4 \sin x}{x} & , x < 0 \\ a - 2x & , x \geq 0 \end{cases}$ is a continuous function, then $a = \underline{\ 4\ }$

3. Let c and L be real numbers, and let f and g be functions such that $\lim\limits_{x \to c} f(x) = \infty$ and

   $\lim\limits_{x \to c} g(x) = -5$. Find the limit $\lim\limits_{x \to c} \dfrac{g(x)}{f(x)} = \underline{\ 0\ }$

4. The equation of the line that is tangent to $f(x) = x^3$ and parallel to $3x - y + 1 = 0$

   is $\underline{\ y - 1 = 3(x - 1) \ \text{ or } \ y + 1 = 3(x + 1)\ }$

5. Let $f(x) = \dfrac{1}{\sqrt{x}}$, then $f'(4) = \underline{\ \dfrac{-1}{16}\ }$

111 學年度第一學期工、電資學院微積分 (3 學分) 第一次會考答案 2022.10.12

題號	答案	來源
1	$(a)\ \dfrac{1}{4},\qquad (b)\ -5$	1.3 — 習題 54*
2	$(a)\ 略,\qquad (b)\ \dfrac{4}{3}$	1.3 — 習題 74,94
3	$(a)\ (-\infty,\infty),\qquad (b)\ (-\infty,-1)\ \text{and}\ (-1,\infty)$	1.4 — 習題 75,31
4	略	1.4 — 例題 8*
5	$x = -2,\ x = 1$	1.5 — 例題 3*
6	$(a)\ 略,\qquad (b)\ 略$	1.4, 2.1 — 定義
7	$(a)\ 4,\qquad (b)\ -\dfrac{1}{x^2}$	2.1 — 例題 1*
8	$(a)\ 略,\qquad (b)\ 0$	2.1 — 習題 97
9	$(a)\ f'(x) = 1 - 8x^{-3},\qquad (b)\ f'(x) = 2x + \dfrac{1}{2}x^{-\frac{1}{2}} - 4\sin x + \dfrac{1}{9}x^{-\frac{8}{9}}$	2.2 — 習題 43,51
10	$y - 2 = -\dfrac{8}{3}(x-1)$	2.2 — 習題 63

* 為非勾選習題、類似題.
證明題過程略過.