中原大學 111 學年度 ■上學期 □下學期 考試命題紙 ■期中考

科目名稱: 微積分 (上)(3 學分)
考試時間: 11 月 9 日第二節

I. 計算、證明題. (80 分)

1. Find $\dfrac{d^2y}{dx^2}$ if $7xy + \sin x = 2$.

2. Prove that $|\sin a - \sin b| \le |a - b|$ for all a and b.

3. Consider $f(x) = 2x - 3x^{\frac{2}{3}}$ on $[-1, 3]$.
   (a) Find the critical number(s) of $f(x)$. (b) Find the absolute extrema of $f(x)$.

4. Use the Quotient Rule to prove $\dfrac{d}{dx}[\csc x] = -\csc x \cot x$.

5. Find the derivative of each of the following functions.
   (a) $g(t) = 5\cos^2(\pi t)$ (b) $f(x) = \left(\dfrac{x+5}{x^2+2}\right)^2$

6. Given $x^2y^2 - 9x^2 - 4y^2 = 1$.
   (a) Find $\dfrac{dy}{dx}$ (b) Find the equation of the tangent line at the point $(4, 2\sqrt{3})$.

7. Determine the point(s) in the interval $(0, 2\pi)$ at which the graph of $f(x) = 2\cos x + \sin 2x$ has a horizontal tangent.

8. Differentiate each of the following functions:
   (a) $g(t) = \sqrt[4]{t} + 6\csc(3t)$ (b) $f(x) = x\left(1 - \dfrac{4}{x+3}\right)$.

9. Find the absolute extrema of the function $g(x) = \dfrac{6x^2}{x-2}$ on the interval $[-2, 1]$.

10. Let $f(x) = \sqrt{2-x}$.
    (a) Can the Mean Value Theorem be applied to $f(x)$ on the interval $[-7, 2]$? Why?
    (b) Find all value(s) of c in $(-7, 2)$ such that $f'(c) = \dfrac{f(-7) - f(2)}{-7 - 2}$.

II. 填充題. (25 分)

1. The $98th$ derivative of $\cos x$ is $\underline{\quad -\cos x \quad}$

2. If $g(5) = -3$ and $g'(5) = 6$, and $f(x) = [g(x)]^3$, then $f'(5) = \underline{\quad 162 \quad}$

3. Let $f(x) = \tan(\sec(\sin x))$, then $f'(x) = \underline{\quad \sec^2(\sec(\sin x))[\tan(\sin x)\sec(\sin x)]\cos x \quad}$

4. The critical number of the function $h(x) = \sin^2 x + \cos x$ in the interval $(\pi, 2\pi)$ is $x = \underline{\quad \frac{5\pi}{3} \quad}$

5. Given $\sin x + 2\cos 2y = 1$. The slope of the tangent line to the graph of this equation at the point $\left(0, \frac{\pi}{6}\right)$ is $\underline{\quad \frac{1}{2\sqrt{3}} \quad}$

111 學年度第一學期工、電資學院微積分 (3 學分) 期中考答案 2022.11.9

題號	答案	來源
1	$\dfrac{14y+2\cos x+x\sin x}{7x^2}$	2.5 − 習題 53
2	略	3.2 − 習題 84
3	(a) $x=0$ and $x=1$, (b) $f(0)=0$ is the absolute maximum value , $f(-1)=-5$ is the absolute minimum value	3.1 − 例題 3
4	略	2.3 − 習題 87(b)*
5	(a) $-10\pi\cos(\pi t)\sin(\pi t)$, (b) $\dfrac{2(x+5)(-x^2-10x+2)}{(x^2+2)^3}$	2.4 − 習題 29.46
6	(a) $y'=\dfrac{18x-2xy^2}{2x^2y-8y}$, (b) 題目錯誤，送分	2.5 − 習題 39
7	$x=\dfrac{3\pi}{2},\ \dfrac{\pi}{6},\ \dfrac{5\pi}{6}$	2.4 − 習題 81*
8	(a) $g'(t)=\dfrac{1}{4}t^{\frac{-3}{4}}-18\csc(3t)\cot(3t)$, (b) $\dfrac{x^2+6x-3}{(x+3)^2}$	2.3 − 習題 47.31
9	$g(1)=-6=g(-2)$ is the absolute minimum value , $g(0)=0$ is the absolute maximum value	3.1 − 習題 31
10	(a)略, $(b)c=-\dfrac{1}{4}$	3.2 − 習題 46*

* 為非勾選習題、類似題.
證明題過程略過.