中原大學 111 學年度 ■上學期 □下學期 考試命題紙■第二次會考

科目名稱: 微積分 (上)(3 學分)

考試時間: 12 月 7 日第二節

I. 計算、證明題. (80 分)

1. (a) Find the open intervals on which $f(x) = (x^2 - 9)^{\frac{2}{3}}$ is increasing or decreasing.
   (b) Find the relative extrema of f.

2. (a) Give a counterexample or prove (給反例或證明) that if $f'(c) = 0$ or $f'(c)$ dose not exist then $(c, f(c))$ is a point of inflection.
   (b) Determine the points of inflection of the graph of $f(x) = x^4 - 4x^3$.

3. Let $f(x) = -5x^7 + 7x^5$.
   (a) Use the First Derivative Test to find the relative extrema of f.
   (b) Use the Second Derivative Test to find the relative extrema of f.

4. (a) Find the open intervals on which the graph of $f(x) = \dfrac{x^2+1}{x^2-1}$ is concave upward or concave downward.
   (b) Determine the point(s) of inflection of f if it (they) exist.

5. (a) Find the limit $\lim\limits_{x\to-\infty} \dfrac{4x-3}{\sqrt{3x^2+2}}$ (b) Find the limit $\lim\limits_{x\to\infty} \left(\sqrt{16x^2-3x} - 4x\right)$.

6. (a) Find the limit $\lim\limits_{x\to\infty} \dfrac{4x - \cos 3x}{4x}$ (b) Find the limit $\lim\limits_{x\to\infty} \sqrt{x}\sin\dfrac{1}{x}$.

7. Find the horizontal, vertical, and slant asymptotes (if they exist) of the function $f(x) = \dfrac{4x^3}{(x+1)^2}$.

8. Sketch the graph of a function that satisfies all of the following conditions:
   (a) $f(0) = 0$
   (b) $\lim\limits_{x\to-3^-} f(x) = \infty$, $\lim\limits_{x\to-3^+} f(x) = -\infty$, $\lim\limits_{x\to3^-} f(x) = -\infty$, $\lim\limits_{x\to3^+} f(x) = \infty$
   (c) $\lim\limits_{x\to\infty} f(x) = 0$, $\lim\limits_{x\to-\infty} f(x) = 0$
   (d) $f'(0) = 0, f'(x) > 0$ for $x < -3$ or $-3 < x < 0$; $f'(x) < 0$ for $x > 3$ or $0 < x < 3$
   (e) $f''(x) > 0$ for $x < -3$ or $x > 3$; $f''(x) < 0$ for $-3 < x < 3$

9. Let $f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{4}{3}}$. Sketch the graph of f by considering the increasing/decreasing intervals, relative extrema, and concavity.

10. Let $y = x^2$.
    (a) Find the differential dy when $x = 1$ and $dx = 0.01$
    (b) Find the change $\triangle y$ when $x = 1$ and $\triangle x = 0.01$.

II. 填充題. (25 分)

1. $\lim_{x\to-\infty} \dfrac{|4x+2|}{x-2} =$ $\underline{\quad -4 \quad}$

2. The relative minimum of $f(x) = x + \sqrt{2}\cos x$ in the interval $(0, 2\pi)$ is at $x =$ $\underline{\quad \dfrac{3\pi}{4} \quad}$

3. The point of inflection of $f(x) = (x-2)(x-1)^2$ occurs at $x =$ $\underline{\quad \dfrac{4}{3} \quad}$

4. Use differentials to estimate $\sqrt[4]{623} \approx$ $\underline{\quad 4.996 \quad}$

5. Let $y = 3x^2 - \sec^2 x$. Then the differential $dy =$ $\underline{\quad (6x - 2\sec^2 x \tan x)\, dx \quad}$

111 學年度第一學期工、電資學院微積分 (3 學分) 第二次會考答案 2022.12.7

題號	答案	來源
1	(a) f is increasing on $(-3,0)$ and $(3,\infty)$, f is decreasing on $(-\infty,-3)$ and $(0,3)$ (b) f has realative maximum $\sqrt[3]{81}$, f has relative minimum 0	$3.3-$ 例題 3^*
2	(a) 略 (b) The points of inflection are $(0,0),(2,-16)$	$3.4-$ 例題 3^*
3	(a) $f(-1)=-2$ is a relative minimum , $f(1)=2$ is a relative maximum (b) $f(-1)=-2$ is a relative minimum , $f(1)=2$ is a relative maximum	$3.4-$ 例題 4^*
4	(a) f is concave upwaed on $(-\infty,-1)$ and $(1,\infty)$, f is concave downward on $(-1,1)$ (b) f are no points of inflection	$3.4-$ 習題 13^*
5	(a) $\dfrac{-4}{\sqrt{3}}$, (b) $\dfrac{-3}{8}$	$3.5-$ 例題 4, 習題 46^*
6	(a) 1, (b) 0	$3.5-$ 習題 $36,41^*$
7	f has no horizontal asymptotes f has a vertical asymptotes $x=-1$ f has a slant asymptotes $y=4x-8$	$3.5-$ 例題 8^*
8	略	3.6
9	略	$3.6-$ 例題 4
10	(a) $dy=f'(x)\,dx=0.02$, (b) $\triangle y=f(x+\triangle x)-f(x)=0.0201$	$3.9-$ 例題 2^*

* 為非勾選習題、類似題.

證明題、做圖題過程略過.