中原大學 111 學年度 ■上學期 □下學期 考試命題紙 ■期末考

科目名稱: 微積分 (上)(3 學分)
考試時間: 1 月 11 日第二節

I. 計算、證明題. (80 分)

1. (6 分) Let $\triangle x_i = \dfrac{1}{n}$ and $c_i = \dfrac{i}{n}$, evaluate the definite integral $\displaystyle\int_0^1 x^2\,dx$ as a limit. (Use the right endpoint)

2. (10 分) Evaluate (a) $\displaystyle\int_0^3 \sqrt{9-x^2}\,dx$. (作圖) (b) $\displaystyle\int_0^2 (2x^2+3x+1)\,dx$.

3. (6 分) Let $g(x) = \displaystyle\int_{-x^2}^{x} t^3\sqrt{1+t^2}\,dt$, find $g'(x)$.

4. (6 分) Differentiate $f(x) = \ln\dfrac{x^2(x^2+1)^3}{\sqrt{2x^4+1}}$.

5. (6 分) Evaluate $\displaystyle\int 2x\sqrt{x-1}\,dx$.

6. (10 分) Evaluate the definite integral
   (a) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^5 x\cos x + \sin x\cos^2 x)\,dx$ (b) $\displaystyle\int_{-3}^{3} x^2(x^2+1)\,dx$.

7. (6 分) Use logarithmic differentiation to find $\dfrac{dy}{dx}$, where $y = \dfrac{x^2\sqrt{3x-2}}{(x+1)^2}, x > \dfrac{2}{3}$.

8. (10 分) Consider the function $F(x)$, where $F(x) = \displaystyle\int \frac{1}{x\ln x}\,dx$.
   (a) Find $F(x)$. (b) Use (a) to show that $\dfrac{d}{dx}F(x) = \dfrac{1}{x\ln x}$.

9. (10 分) Evaluate the integrals (a) $\displaystyle\int \frac{x}{x^2+1}\,dx$ (b) $\displaystyle\int \frac{2x^2+5x+5}{2x+3}\,dx$.

10. (10 分) Let $f(x) = 5-2x^3$. (a) Find $f^{-1}(x)$ (b) Find $(f^{-1})'(7)$.

II. 填充題. (25 分)

1. Find the area of the region bounded by the graph of $y = |2x + 1|$ the $x$-axis, and the vertical lines
   $x = 0$ and $x = 2$. Area $=$ $\underline{6}$

2. Let $f(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, find $f'(2) =$ $\underline{3}$

3. If $\int_{-1}^{1} f(x)\, dx = 3$, and $\int_0^1 f(x)\, dx = 5$, then $\int_{-1}^{0} 2f(x)\, dx =$ $\underline{-4}$

4. $\int_0^{\frac{\pi}{4}} \tan x\, dx =$ $\underline{\frac{1}{2}\ln 2}$

5. Let $f(x) = x^2, x > 0$. Find $(f^{-1})'(1) =$ $\underline{\frac{1}{2}}$

# 111 學年度第一學期工、電資學院微積分 (3 學分) 期末考答案 2023.1.11

| 題號 | 答案 | 來源 |
|---|---|---|
| 1 | $\frac{1}{3}$ | 4.3 – 習題 3* |
| 2 | $(a)$ 略, $(b)\ \frac{40}{3}$ | 4.3 – 例題 3* |
| 3 | $g'(x) = x^3\sqrt{1+x^2} - 2x^7\sqrt{1+x^4}$ | 4.4 – 例題 |
| 4 | $f'(x) = \frac{2}{x} + \frac{6x}{x^2+1} - \frac{4x^3}{2x^4+1}$ | 5.1 – 例題 5* |
| 5 | $\frac{4}{5}(x-1)^{\frac{5}{2}} + \frac{4}{3}(x-1)^{\frac{3}{2}} + C$ | 4.5 – 例題 5* |
| 6 | $(a)\ 0, \quad (b)\ \frac{576}{5}$ | 4.5 – 習題 10.73* |
| 7 | $\frac{dy}{dx} = \frac{x^2\sqrt{3x-2}}{(x+1)^2}\left(\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{x+1}\right)$ | 5.1 – 習題 77 |
| 8 | $(a)\ F(x) = \ln\lvert\ln x\rvert + C, \quad (b)$ 略 | 5.1 – 例題* |
| 9 | $(a)\ \frac{1}{2}\ln(x^2+1) + C, \quad (b)\ \frac{x^2}{2} + x + \ln\lvert 2x+3\rvert + C$ | 5.2 – 例題 |
| 10 | $(a)\ f^{-1}(x) = \sqrt[3]{\frac{5-y}{2}}, \quad (b)\ (f^{-1})'(7) = -\frac{1}{6}$ | 5.3 – 習題 |

* 為非勾選習題、類似題.

證明題、做圖題過程略過.