

科目名稱: 微積分(下)(3學分)

考試時間: 4月12日第二節

I. 填充題. (25分)

1.  $\int_0^{\frac{\pi}{4}} \sin^2 x \, dx = \frac{\pi - 2}{8}$

2.  $\int_0^2 \frac{1}{(x^2 + 1)^{\frac{3}{2}}} \, dx = \frac{2\sqrt{5}}{5}$

3.  $\int \frac{1}{9 - x^2} \, dx = \lambda \ln \left| \frac{3 + x}{3 - x} \right| + C$ , where  $\lambda = \frac{1}{6}$

4.  $\int_1^{\infty} \left( \frac{2}{x} \right)^3 \, dx = 4$

5. The limit  $\lim_{(x,y) \rightarrow (0,0)} \frac{2x^3y}{x^6 + y^2}$  along the curve  $y = x^3$  is 1

II. 計算、證明題. (80 分)

1. Find  $\int \tan^3(2t) \sec^3(2t) dt$ .

2. Find  $\int \frac{3-x}{3x^2-2x-1} dx$ .

3. Determine whether the improper integral diverges or converges.

Evaluate the integral if it converges. (a)  $\int_1^{\infty} \frac{3}{\sqrt[3]{x}} dx$ . (b)  $\int_0^1 \frac{3}{\sqrt[3]{x}} dx$ .

4. Evaluate the indefinite integral (a)  $\int \frac{\cos^5 t}{\sqrt{\sin t}} dt$ . (b)  $\int \frac{\sec x}{\tan^2 x} dx$ .

5. Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{4y^2x}{x^2+y^2}$  if it exists.

6. Evaluate  $\int_0^1 x \ln x dx$  if it converges.

7. Evaluate  $\int_{\sqrt{3}}^2 \frac{\sqrt{x^2-3}}{x} dx$ .

8. Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+y^2}{xy}$  does not exist.

9. Evaluate  $\int \frac{x}{\sqrt{4x-x^2}} dx$ .

10. Evaluate  $\int \frac{x-1}{x^2(x+1)} dx$ .

## 111 學年度第 2 學期工、電資學院微積分 (3 學分) 期中考答案 2023.4.12

題號	答案	來源
1	$\frac{1}{10} \left( \frac{1}{\cos 2t} \right)^5 - \frac{1}{6} \left( \frac{1}{\cos 2t} \right)^3 + C$	8.3 - 習題 27
2	$\frac{1}{2} \ln  x - 1  - \frac{5}{6} \ln  3x + 1  + C$	8.5 - 習題 6
3	(a) $\infty$ , (b) $\frac{9}{2}$	8.8 - 習題 19, 例題 6
4	(a) $2(\sin t)^{\frac{1}{2}} - \frac{4}{5}(\sin t)^{\frac{5}{2}} + \frac{2}{9}(\sin t)^{\frac{9}{2}} + C$ , (b) $-\csc x + C$	8.3 - 習題 10, 例題 7
5	略	13.2 - 例題 3*
6	$-\frac{1}{4}$	8.8 - 習題 37
7	$\sqrt{x^2 - 3} - \sqrt{3} \operatorname{arcsec} \left( \frac{x}{\sqrt{3}} \right) + C$	8.4 - 例題 4
8	略	13.2 - 習題 47
9	$2 \arcsin \left( \frac{x-2}{2} \right) - \sqrt{4 - (x-2)^2} + C$	8.4 - 習題 33
10	$2 \ln  x  + \frac{1}{x} - 2 \ln  x+1  + C$	8.5 - 習題 22

\* 為非勾選習題、類似題。

證明題過程略過。