中原大學 111 學年度 □上學期 ■下學期 考試命題紙■第二次會考

科目名稱: 微積分 (下)(3 學分)
考試時間: 5 月 17 日第二節

I. 填充題. (25 分)

1. Let $f(x,y,z) = x^2e^y + zx$, then $f_{xzy}(1,2,3) = \underline{\ 0\ }$

2. Let $z = 3xy + y^2$, where $x = \cos t, y = \sin t$, then $\left.\dfrac{dz}{dt}\right|_{t=0} = \underline{\ 3\ }$

3. Let $f(x,y,z) = 2x^2 + 3y^2 - z$, and **u** be any unit vector. The maximum value of

   $D_{\mathbf{u}}f(1,0,0) = \underline{\ \sqrt{17}\ }$

4. Let θ be the angle of inclination of the tangent plane to the surface $x^2 - y^2 + z = 0$

   at the point $(1,2,3)$, then $\cos\theta = \underline{\ \dfrac{1}{\sqrt{21}}\ }$

5. Find all critical points of then function $f(x,y) = x^2 + 4xy + 2y^2 + 6x - 4y + 3$.

   Ans = $\underline{\ (5,-4)\ }$

II. 計算、證明題. (80 分)

1. Let $f(x,y,z)=e^{-x}\sin yz$, find f_{xy} and f_{yz}.

2. Find $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$, when $s=2, t=\pi$ for $w=x+y^2z+xz^2$, where $x=s\cos t$, $y=s\sin t$ and $z=t$.

3. Find $\dfrac{\partial w}{\partial z}$ and $\dfrac{\partial y}{\partial w}$ for $\cos xy+\sin yz+wz=20$.

4. Find the directional derivative of $f(x,y)=2x^2+3y^2$ at $(0,2)$ in the direction from $P(0,2)$ to $Q\left(\dfrac{3}{4},3\right)$

5. Let $f(x,y)=\dfrac{x+y}{y+1}$, and $\mathbf{u}$ be the unit vector maximizing the value of $D_{\mathbf{u}}f(x,y)$ at the point $(0,1)$. Find $\mathbf{u}$ and $D_{\mathbf{u}}f(x,y)$.

6. Find the tangent plane and the normal line to the elliptic paraboloid $z=2x^2+y^2$ at the point $(1,1,3)$.

7. Find a set of symmetric equations for the normal line and the tangent plane to the surface $xyz=12$ at the point $(2,-2,-3)$.

8. Find a set of parametric equations for the tangent line to the curve of intersection of $x^2+z^2=25$ and $y^2+z^2=25$ at the point $(3,3,4)$.

9. Find the relative extrema of $f(x,y)=-x^3+4xy-2y^2+1$ by the second partials test.

10. Find the absolute extrema of the function $f(x,y)=x^2-4xy+5$ over the region $R=\{(x,y):1\le x\le 4,\ 0\le y\le 2\}$.

111 學年度第 2 學期工、電資學院微積分 (3 學分) 第二次會考答案 2023.5.17

題號	答案	來源
1	$f_{xy} = -ze^{-x}\cos(yz), f_{yz} = e^{-x}\cos(yz) - yze^{-x}\sin(yz)$	13.3 – 習題 93*
2	$\frac{\partial w}{\partial s}(2,\pi) = -\pi^2 - 1, \frac{\partial w}{\partial t}(2,\pi) = -4\pi$	13.5 – 例題 5*
3	$\frac{\partial w}{\partial z} = -\frac{-y\cos(yz)+w}{z},\ \frac{\partial y}{\partial w} = \frac{-z}{z\cos(yz) - x\sin(xy)}$	13.5 – 習題 31
4	$\frac{48}{5}$	13.6 – 例題 4
5	$D_{\mathbf{u}}f(x,y) = \frac{\sqrt{5}}{4},\ \mathbf{u} = (\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$	13.6 – 習題 30*
6	The tangent plane: $4x + 2y - z - 3 = 0$, The normal line: $\frac{x-1}{4} = \frac{y-1}{2} = \frac{z-3}{-1}$	13.7 – 習題?*
7	The tangent plane: $6x - 6y - 4z - 36 = 0$, The normal line: $\frac{x-2}{6} = \frac{y+2}{-6} = \frac{z+3}{-4}$	13.7 – 例題 4
8	The tangent line: $\frac{x-3}{4} = \frac{y-3}{4} = \frac{z-4}{-3}$	13.7 – 習題 29
9	$(\frac{4}{3}, \frac{4}{3}, \frac{59}{27})$ is a relative maximum.	13.8 – 例題 3
10	$f(x,y)$ has absolute maximum at (4,0) with value 21, absolute minimum at (4,2) with values -11	13.8 – 習題 39

* 為非勾選習題、類似題.
證明題過程略過.