

科目名稱: 微積分(下)(3學分)

考試時間: 6月14日第二節

I. 填充題. (25分)

1. $\int_0^2 \int_0^1 (x+y) dy dx = \underline{3}$

2. Evaluate $\iint_R 4xy dA$, where R is a rectangle with vertices $(0,0), (0,5), (3,5), (3,0)$.

Ans = 225

3. $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 5(x^2+y^2)^{\frac{3}{2}} dy dx = \underline{2\pi}$

4. $\int_0^1 \int_0^{2\pi} r \sin(r\theta) d\theta dr = \underline{1}$

5. Find the 3rd Taylor polynomial for $\sin x + \cos x$ at $c = 0$.

Ans = $1 + x - \frac{x^2}{2} - \frac{x^3}{6}$

II. 計算、證明題. (80 分)

1. (6 分) $\int_0^{\ln 3} \int_0^{\ln 2} e^{x+y} dy dx.$

2. (8 分) $\int_0^1 \int_{2x}^2 4e^{y^2} dy dx.$

3. (8 分) $\int_0^1 \int_0^1 \sqrt{|y-x|} dx dy.$

4. (8 分) Find the volume of the solid region bounded by the plane $x + 2y + z = 2$, and xy, xz, yz planes.

5. (6 分) Find $\iint_R \frac{x}{\sqrt{1+y^2}} dA$, where R is the region enclosed by $y = x^2, y = 4$ and $x = 0$ in the first quadrant (第一象限).

6. (8 分) Find the volume of the solid region bounded by the surface $f(x, y) = 2e^{-x^2}$ and the planes $z = 0, y = 0, y = x$ and $x = 1$.

7. (8 分) Find the volume of the solid region lying between the surface $z = \sqrt{9 - x^2 - y^2}$ and the circular region $x^2 + y^2 \leq 1$ on the xy plane.

8. (8 分) $\int_0^3 \int_0^{\sqrt{9-x^2}} e^{-x^2-y^2} dy dx.$

9. (10 分)

(a) Write down the 3rd Taylor polynomial for e^x at $c = 0$.

(b) Use (a) to obtain the 3rd Taylor polynomial for e^{-x^2} at $c = 0$.

(c) Use (b) to approximate(近似) $\int_0^1 e^{-x^2} dx$.

10. (10 分)

(a) Write down the 4th Taylor polynomial for $\ln(1+x)$ at $c = 0$.

(b) Use the 4th Taylor polynomial from (a) to approximate(近似) $\ln 2$.

(c) Use (a) and (b) to further approximate(近似) $\ln(8.8)$.

111 學年度第 2 學期工、電資學院微積分 (3 學分) 期末考答案 2023.6.14

題號	答案	來源
1	2	14.1 - 習題 14*
2	$e^4 - 1$	14.1 - 習題 63
3	$\frac{8}{15}$	14.1 - 習題
4	$\frac{2}{3}$	14.2 - 習題 23*
5	$\frac{1}{2}(\sqrt{17} - 1)$	14.2 - 習題 15
6	$1 - e^{-1}$	14.2 - 例題 4*
7	$\frac{2\pi}{3}(27 - 16\sqrt{2})$	14.3 - 例題 3*
8	$\frac{\pi}{4}(1 - e^{-9})$	14.3 - 習題 30
9	(a) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ (b) $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!}$ (c) $1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42}$	泰勒講義
10	(a) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$ (b) $\frac{7}{12}$ (c) $\frac{7}{4} + (0.1) - \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} - \frac{(0.4)^4}{4}$	泰勒講義

* 為非勾選習題、類似題.

證明題過程略過.