科目名稱: 微積分 (下)(3 學分)
考試時間: 6 月 14 日第二節

I. 填充題. (25 分)

1. $\displaystyle\int_0^2 \int_0^1 (x+y)\, dy dx = \underline{\;3\;}$

2. Evaluate $\displaystyle\iint_R 4xy\, dA$, where $R$ is a rectangle with vertices $(0,0), (0,5), (3,5), (3,0)$.

   Ans $= \underline{\;225\;}$

3. $\displaystyle\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 5(x^2+y^2)^{\frac{3}{2}}\, dy dx = \underline{\;2\pi\;}$

4. $\displaystyle\int_0^1 \int_0^{2\pi} r \sin(r\theta)\, d\theta dr = \underline{\;1\;}$

5. Find the $3rd$ Taylor polynomial for $\sin x + \cos x$ at $c=0$.

   Ans $= \underline{\; 1 + x - \dfrac{x^2}{2} - \dfrac{x^3}{6} \;}$

II. 計算、證明題. (80 分)

1. (6 分) $\displaystyle\int_0^{\ln 3}\int_0^{\ln 2} e^{x+y}\,dydx$.

2. (8 分) $\displaystyle\int_0^1\int_{2x}^2 4e^{y^2}\,dydx$.

3. (8 分) $\displaystyle\int_0^1\int_0^1 \sqrt{|y-x|}\,dxdy$.

4. (8 分) Find the volume of the solid region bounded by the plane $x + 2y + z = 2$, and $xy, xz, yz$ planes.

5. (6 分) Find $\displaystyle\iint_R \frac{x}{\sqrt{1+y^2}}\,dA$, where $R$ is the region enclosed by $y = x^2, y = 4$ and $x = 0$ in the first quadrant (第一象限).

6. (8 分) Find the volume of the solid region bounded by the surface $f(x,y) = 2e^{-x^2}$ and the planes $z = 0, y = 0, y = x$ and $x = 1$.

7. (8 分) Find the volume of the solid region lying between the surface $z = \sqrt{9 - x^2 - y^2}$ and the circular region $x^2 + y^2 \le 1$ on the $xy$ plane.

8. (8 分) $\displaystyle\int_0^3\int_0^{\sqrt{9-x^2}} e^{-x^2-y^2}\,dydx$.

9. (10 分)
   (a) Write down the $3rd$ Taylor polynomial for $e^x$ at $c = 0$.
   (b) Use (a) to obtain the $6rd$ Taylor polynomial for $e^{-x^2}$ at $c = 0$.
   (c) Use (b) to approximate(近似) $\displaystyle\int_0^1 e^{-x^2}\,dx$.

10. (10 分)
    (a) Write down the $4th$ Taylor polynomial for $\ln(1+x)$ at $c = 0$.
    (b) Use the $4th$ Taylor polynomial from (a) to approximate(近似) $\ln 2$.
    (c) Use (a) and (b) to further approximate(近似) $\ln(8.8)$.

# 111 學年度第 2 學期工、電資學院微積分 (3 學分) 期末考答案 2023.6.14

| 題號 | 答案 | 來源 |
|---|---|---|
| 1 | $2$ | 14.1 — 習題 14* |
| 2 | $e^4 - 1$ | 14.1 — 習題 63 |
| 3 | $\dfrac{8}{15}$ | 14.1 — 習題 |
| 4 | $\dfrac{2}{3}$ | 14.2 — 習題 23* |
| 5 | $\dfrac{1}{2}(\sqrt{17} - 1)$ | 14.2 — 習題 15 |
| 6 | $1 - e^{-1}$ | 14.2 — 例題 4* |
| 7 | $\dfrac{2\pi}{3}(27 - 16\sqrt{2})$ | 14.3 — 例題 3* |
| 8 | $\dfrac{\pi}{4}(1 - e^{-9})$ | 14.3 — 習題 30 |
| 9 | $(a)\,1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!}$  $(b)\,1 - x^2 + \dfrac{x^4}{2!} - \dfrac{x^6}{3!}$  $(c)\,1 - \dfrac{1}{3} + \dfrac{1}{10} - \dfrac{1}{42}$ | 泰勒講義 |
| 10 | $(a)\,x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4}$  $(b)\,\dfrac{7}{12}$  $(c)\,\dfrac{7}{4} + (0.1) - \dfrac{(0.1)^2}{2} + \dfrac{(0.1)^3}{3} - \dfrac{(0.4)^4}{4}$ | 泰勒講義 |

\* 為非勾選習題、類似題.

證明題過程略過.