中原大學 112 學年度 ■上學期 □下學期 考試命題紙 ■期中考

科目名稱: 微積分 (上)(A 群)
考試時間: 11 月 8 日第二節

I. 填充題. (25 分)

1. Let $f(x) = \sin(\cos(x^2))$, then $f'(x) =$ $-2x\cos(\cos x^2)\sin x^2$

2. The slope of the graph of $x^2 + y^2 = 4$ at the point $(1, \sqrt{3})$ is $-\frac{1}{\sqrt{3}}$

3. The critical number(s) of $f(x) = 2x - 3x^{\frac{2}{3}}$ are 0 and 1

4. Let $f(x) = x\cos x$, then $f''(x) =$ $-2\sin x - x\cos x$

5. Let g be differentiable and $f(x) = g(2xg(x))$ with $g(0) = 1$ and $g'(0) = 3$, then $f'(0) =$ 6

II. 計算、證明題. (80 分)

1. (6%) Find the derivative of the given functions. (a) $f(x) = \dfrac{5x-2}{x^2+1}$ (b) $g(x) = x^2 \tan x$

2. (6%) Find the tangent line to the graph of $f(x) = \dfrac{x}{x+4}$ at the point $(-3, -3)$.

3. (10%) Find $\dfrac{dy}{dx}$ of the given functions by implicit differentiation.
   (a) $x^3 - xy + y^2 = 7$ (b) $y = \sin xy$

4. (10%) Find the absolute extrema of $f(x) = 3x^4 - 4x^3$ on the closed interval $[-1, 2]$.

5. (10%) Find the absolute extrema of $\dfrac{6x^2}{x-2}$ on the closed interval $[-2, 1]$.

6. (10%) Prove that $|\sin a - \sin b| \leq |a - b|$, for all a and b with $a \neq b$.

7. (6%) Let $f(x) = x^4 - 3x^3 + 5$. Prove that there is a number $c \in (0, 2)$ such that the slope of the tangent line on $f(x)$ that goes through $(c, f(c))$ is -4.

8. (6%) Find the derivative of each of the following functions.
   (a) $f(t) = \sin^3(4t)$. (b) $g(x) = \left(\dfrac{3x^2-2}{2x+3}\right)^{-2}$

9. (6%) Determine the point(s) at which the graph of $f(x) = \dfrac{-4x}{\sqrt{2x-1}}$ has a horizontal tangent.

10. (10%) Given $\tan(x+y) = x$. (a) Find $\dfrac{dy}{dx}$. (b) Find the equation of the tangent line at the point $(0, 0)$.

112 學年度第一學期理、工、電資學院微積分 (A 群) 期中考答案 2023.11.8

題號	答案	來源
1	$(a)\ f'(x)=\dfrac{-5x^2+4x+5}{(x^2+1)^2}$ $(b)\ g'(x)=2x\tan x+x^2\sec^2 x$	2.3 − 例題 4, 習題 53
2	$y+3=4(x+3)$	2.3 − 習題 65*
3	$(a)\ y'=\dfrac{y-3x^2}{-x+2y}$ $(b)\ y'=\dfrac{y\cos(xy)}{1-x\cos(xy)}$	2.5 − 習題 9,19
4	absolute minimum is -1, absolute maximum is 16.	3.1 − 例題 2
5	absolute minimum is -6, absolute maximum is 0.	3.1 − 習題 31
6	略	3.2 − 習題 84
7	略	3.2 − 習題 37
8	$(a)\ f'(t)=12\sin^2(4t)\cos(4t)$ $(b)\ g'(x)=\dfrac{-2(2x+3)(6x^2+18x+4)}{(3x^2-2)^3}$	2.4 − 例題 12, 習題 32
9	The point is $(1,-4)$.	2.4 − 習題 80*
10	$(a)\ \dfrac{dy}{dx}=\dfrac{1-\sec^2(x+y)}{\sec^2(x+y)}$ $(b)\ y=0$	2.5 − 習題 31

* 為非勾選習題、勾選習題類似題.
證明題過程略過.