中原大學 112 學年度 ■上學期 □下學期 考試命題紙 ■期中考

科目名稱: 微積分 (上)(B 群)
考試時間: 11 月 8 日第二節

I. 填充題. (25 分)

1. Let $f(x) = 2x^{\frac{3}{2}} + x + 9$. Find $f'(16) =$ 13

2. Let $f(x) = x^{\frac{1}{2}}(x^2 + 3x - 1)$. Find $f'(x) =$ $\frac{5}{2}x^{\frac{3}{2}} + \frac{9}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$

3. Let $y = \dfrac{\sin x}{1 + \cos x}$. Find $y' =$ $\dfrac{1}{1 + \cos x}$

4. Let $F(x) = (x^2 + x + 1)^{100}$. Find $F'(x) =$ $100(2x + 1)(x^2 + x + 1)^{99}$

5. Let $g(t) = \tan(\sin 2t)$. Find $g'(t) =$ $2\cos 2t \sec^2(\sin 2t)$

II. 計算、證明題. (80 分)

1. Let $y = \dfrac{x^3 - 2x^2 + x - 4}{2\sqrt{x}}$. Find y'.

2. Let $g(x) = (x^2 + 1)f(x)$ with $f(2) = 3$ and $f'(2) = -1$. Find $g'(2)$.

3. Let $f(x) = 3x^5 + x^4 - 3x^3 + 8x - 9$. Find $f'(1)$.

4. Let $f(x) = \dfrac{x^2 + 2x}{x^3 - 1}$. Find $f'(x)$.

5. Let $f(x) = 8x^7 - 6x^5 + 4x^3 - x$. Find $f'''(0)$.

6. Let $y = \sin x - 2\cos x$. Find $y'|_{x=\frac{\pi}{6}}$.

7. Let $y = (\sec x)(x + \tan x)$. Find y'.

8. Let $y = \tan^3(3x^2 + 1)$. Find $\dfrac{dy}{dx}$.

9. Find the tangent line of $f(x) = x \sin \dfrac{1}{x}$ at the point $\left(\dfrac{2}{\pi}, \dfrac{2}{\pi}\right)$.

10. Suppose that $F(x) = g(f(x))$, and $f(3) = 16$, $f'(3) = 6$ and $g'(16) = \dfrac{1}{8}$. Find $F'(3)$.

112 學年度第一學期理、工、電資學院微積分 (B 群) 期中考答案 2023.11.8

題號	答案	來源
1	$y' = \frac{5}{4}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{4}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$	2.2 – 例題 6
2	$g'(2) = 7$	2.3 – 例題 3
3	$f'(1) = 18$	2.2 – 例題 5*
4	$f'(x) = \frac{-x^4 - 4x^3 - 2x - 2}{(x^3 - 1)^2}$	2.3 – 例題 4*
5	$f'''(0) = 24$	2.3 – 習題 28
6	$y' \mid_{x=\frac{\pi}{6}} = \frac{\sqrt{3}}{2} + 1$	2.4 – 習題 17*
7	$y' = \sec x \tan x(x + \tan x) + \sec x(1 + \sec^2 x)$	2.4 – 例題 2
8	$\frac{d}{dx} = 18x \tan^2(3x^2 + 1) \sec^2(3x^2 + 1)$	2.5 – 例題 8
9	$y = x$	2.5 – 習題 22*
10	$F'(3) = \frac{3}{4}$	2.5 – 習題 31

* 為非勾選習題、勾選習題類似題.
證明題過程略過.