中原大學 112 學年度 ■上學期 □下學期 考試命題紙■第一次會考

科目名稱: 微積分 (上)(A 群)
考試時間: 10 月 11 日第二節

I. 填充題. (25 分)

1. Find $\lim_{x\to 0} \dfrac{1-\cos x}{2x^2} =$ $\dfrac{1}{4}$

2. If $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & , x \neq 2 \\ c & , x = 2 \end{cases}$ is a continuous function, then $c =$ 4

3. Find $\lim_{x\to 0^+} \dfrac{\sin x + \cos x}{x - \frac{1}{\sqrt{x}} + 1} =$ 0

4. Find $\lim_{\triangle x\to 0} \dfrac{[5-3(1+\triangle x)^2]-2}{\triangle x} =$ -6

5. Let $f(x) = \dfrac{x^3-3x^2+4}{x^2}$, then $f'(2) =$ 0

II. 計算、證明題. (80 分)

1. (5 分) (a) Find $\lim_{x\to 1}\dfrac{x^3-1}{x-1}$ (5 分) (b) Find $\lim_{x\to 0}\dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

2. (5 分) (a) Find $\lim_{x\to 0}\dfrac{\sin 4x}{\sin 3x}$ (5 分) (b) Find $\lim_{x\to 0}\dfrac{\tan^2 x}{x}$

3. (6 分) Let $[[x]]$ be the greatest integer n such that $n \le x$.
   Let $f(x) = [[\dfrac{x}{3}]] + 3$. Is f continuous at $x = 3$? Explain your answer.

4. (6 分) Let $f(x) = \dfrac{2}{\pi^2}x^2 - 2 - \sin x - \cos x$.
   Show that there is a number c in $[0, \pi]$ such that $f(c) = 0$.

5. (8 分) Let $h(t) = \dfrac{t^2-2t}{t^4-16}$. Find the vertical asymptotes of h. Explain your answer.

6. (8 分) Find the derivative of $f(x) = \sqrt{x+2}$ for $x > -2$ by the limit process.

7. (5 分) (a) Show that $\lim_{x\to 0} x \sin\dfrac{1}{x} = 0$.
   (5 分) (b) Let $g(x) = \begin{cases} x^2 \sin\dfrac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$. Find $g'(0)$.

8. (8 分) Is $f(x) = \begin{cases} x^2+1 & , x \le 2 \\ 4x-3 & , x > 2 \end{cases}$ differentiable at $x = 2$? Explain your answer.

9. (6 分) Find the tangent line of $f(x) = \dfrac{5}{\sqrt[5]{x^3}}$ at point $(1, 5)$.

10. (8 分) Find the tangent line to $f(x) = x^3$ that is parallel to $27x - y + 5 = 0$.

112 學年度第一學期理、工、電資學院微積分 (A 群) 第一次會考答案 2023.10.11

題號	答案	來源
1	$(a)\ 3, \quad (b)\ \dfrac{1}{2\sqrt{2}}$	1.3 – 例題 6, 習題 56*
2	$(a)\ \dfrac{4}{3}, \quad (b)\ 0$	1.3 – 習題 74, 68*
3	略	1.4 – 習題 27*
4	略	1.4 – 例題 95*
5	$x = -2$ is the only asymptote of h	1.5 – 習題 28
6	$\dfrac{1}{2\sqrt{x+2}}$	2.1 – 習題 27*
7	(a) 略, $\quad (b)\ g'(0) = 0$	2.1 – 習題 99
8	略	2.1 – 習題 91
9	$y = -3x + 8$	2.2 – 習題 57*
10	Tangent line: $y - 27 = 27(x - 3)$ and $y + 27 = 27(x + 3)$	2.1 – 習題 40*

* 為非勾選習題、勾選習題類似題.
證明題過程略過.