中原大學 112 學年度 ■上學期 □下學期 考試命題紙■第二次會考

科目名稱: 微積分 (上)(A 群)
考試時間: 12 月 6 日第二節

I. 填充題. (25 分)

1. Let $y = \sin 3x$. Find $dy =$ $3\cos(3x)\,dx$

2. The relative maximum of $f(x) = (x^2 - 4)^{\frac{2}{3}}$ occurs at $x =$ 0

3. The two points of inflection of $f(x) = x^4 - 4x^3 + 8$ at $x =$ $0, 2$

4. Find $\lim_{x \to -\infty} \dfrac{|3x + 2|}{x - 2} =$ -3

5. The tangent line approximation of $f(x) = \dfrac{1}{2} - \sin x$ at $\left(\dfrac{\pi}{6}, 0\right)$ is $y =$ $-\dfrac{\sqrt{3}}{2}\left(x - \dfrac{\pi}{6}\right)$

II. 計算、證明題. (80 分)

1. (a) Find the open intervals on which $f(x) = \dfrac{x^4+1}{x^2}$ is increasing or decreasing.

   (b) Find the relative extrema of f.

2. (a) Determine the open intervals on which the graph of $f(x) = \dfrac{6}{x^2+3}$ is concave upward or concave downward. (b) Find the relative extrema of f.

3. Let $f(x) = x\sqrt{16-x^2}$. (a) Find the open intervals on which the function $f(x)$ is increasing or decreasing. (b) Find all relative extrema.

4. Let $f(x) = x + \dfrac{4}{x}$. (a) Find all relative extrema. (b) Find all points of inflection, if any.

5. Find the limits (a) $\lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2-x}}$ (b) $\lim\limits_{x\to\infty} \dfrac{2x^2+x}{6x^3+2x^2+x}$.

6. Find the limits (a) $\lim\limits_{x\to\infty} \dfrac{\sin x}{x^2+1}$ (b) $\lim\limits_{x\to-\infty} (3x+\sqrt{9x^2-x})$.

7. Let $f(x) = \dfrac{2x}{\sqrt{x^2+2}}$. Find the horizontal asymptotes of function $f(x)$.

8. Use differentials to approximate (a) $\sqrt{99.4}$ and (b) $\tan(0.05)$.

9. Let $f(x) = 3x^4 + 4x^3$.
   (a) Find the critical numbers of $f(x)$.
   (b) Find the relative extrema of $f(x)$.
   (c) Find the points of inflection of $f(x)$.
   (d) Find the asymptotes of $f(x)$.
   (e) Sketch the graph of $f(x)$.

10. Let $f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{4}{3}}$, $f'(x) = \dfrac{10}{3}\sqrt[3]{x}(\sqrt[3]{x}-2)$, $f''(x) = \dfrac{20}{9}\dfrac{\sqrt[3]{x}-1}{\sqrt[3]{x^2}}$.

    Sketch the graph of f by considering the increasing/ decreasing intervals, relative extrema and concavity.

112 學年度第一學期理、工、電資學院微積分 (A 群) 第二次會考答案 2023.12.6

題號	答案	來源
1	(a) f is increasing on $(-1,0)$ and $(1,\infty)$, decreasing on $(-\infty,-1)$ and $(0,1)$ (b) $f(1)=f(-1)=2$ is relative minimum	3.3 – 例題 4
2	(a) f is concave upward on $(-\infty,-1),(1,\infty)$, concave downward on $(-1,1)$ (b) $f(0)=2$ is relative maximum	3.4 – 例題 1
3	(a) $(-4,-2\sqrt{2}),(2\sqrt{2},4)$ is dereasing interval, $(-2\sqrt{2},2\sqrt{2})$ increasing interval (b) $f(-2\sqrt{2})=-8$ is relative minimum, $f(2\sqrt{2})=8$ is relative maximum	3.3 – 習題 13
4	(a) $f(-2)=-4$ is relative minimum, $f(2)=4$ is relative maximum (b) no points of inflection	3.4 – 習題 23*
5	(a) -1, (b) 0	3.5 – 習題 21,25
6	(a) 0, (b) $\frac{1}{6}$	3.5 – 習題 41,45*
7	The horizontal asymptotes are $y=2, y=-2$	3.5 – 習題 6
8	(a) 9.97 (b) 0.05	3.9 – 習題 43,48
9	(a) $x=0,1$ (b) $f(-1)=-1$ is relative minimum (c) point of inflection $\left(-\frac{2}{3},-\frac{16}{27}\right),(0,0)$ (d) None asymptote (e) 略	3.6 – 習題 33
10	略	3.6 – 例題 4

* 為非勾選習題、勾選習題類似題.
證明題過程略過.