中原大學 112 學年度 □上學期 ■下學期 考試命題紙 ■期末考

科目名稱: 微積分 (上)(A 群)
考試時間: 1 月 10 日第二節

I. 填充題. (25 分)

1. If $\lim_{n\to\infty} \frac{1}{n^{\frac{5}{3}}} \left(\sqrt[3]{1^2} + \sqrt[3]{2^2} + ... + \sqrt[3]{n^2}\right) = \int_0^1 x^r \, dx$, then $r =$ $\frac{2}{3}$

2. Find $\int_{-6}^{0} \sqrt{36 - x^2} \, dx =$ 9π

3. If $\int_0^1 f(x) \, dx = 1$ and $\int_2^1 f(x) \, dx = 3$, then $\int_2^0 f(x) \, dx =$ 2

4. Find $\frac{d}{dx} \int_{\sqrt{2}}^{x} \frac{t^2}{t^4 + 1} \, dt =$ $\frac{x^2}{x^4 + 1}$

5. If $f(x) = x^3 + x - 1$, then $(f^{-1})'(1) =$ $\frac{1}{4}$

II. 計算、證明題. (80 分)

1. Evaluate $\int_0^4 |x^2 - 9|\, dx$.

2. Find the area of the region bounded by the graphs of the equations $y = 1+\sqrt[3]{x}, x = 0, x = 8$ and $y = 0$.

3. Find the derivative of $F(x) = \int_{x^4}^{x^3} \sin t^2\, dt$.

4. Find $\int \left(\sin^3 x \cos x + \sin x \cos x\right)\, dx$.

5. Find $\int x\sqrt{2x-1}\, dx$.

6. Use the Logarithmic Differentiation to find the derivative of $y = \dfrac{x^2\sqrt{3x-2}}{(x+1)^2}, x > \dfrac{2}{3}$.

7. Find $\int \dfrac{x^2+x+1}{x^2+1}\, dx$.

8. Evaluate $\int_0^{\frac{\pi}{4}} \sqrt{1+\tan^2 x}\, dx$.

9. Evaluate $\int_e^{e^2} \dfrac{1}{x \ln x}\, dx$.

10. Show that $f(x) = \int_2^x \dfrac{1}{\sqrt{1+t^2}}\, dt$ is increasing on $(-\infty, \infty)$, and find $(f^{-1})'(0)$.

112 學年度第一學期理、工、電資學院微積分 (A 群) 期末考答案 2024.1.10

題號	答案
1	$\frac{64}{3}$
2	20
3	$[\sin(x^3)^2](3x^2) - [\sin(x^4)^2](4x^3)$
4	$\frac{1}{4}\sin^4 x + \frac{1}{2}\sin^2 x + C$ or $\frac{1}{4}\cos^4 x - \cos^2 x + C$
5	$\frac{1}{10}(2x-1)^{\frac{5}{2}} + \frac{1}{6}(2x-1)^{\frac{3}{2}} + C$
6	$y' = \frac{x^2\sqrt{3x-2}}{(x+1)^2}\left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{x+1}\right]$
7	$x + \frac{1}{2}\ln\lvert x^2+1\rvert + C$
8	$\ln(\sqrt{2}+1)$
9	$\ln 2$
10	證明部分略過, $(f^{-1})'(0) = \sqrt{5}$

* 為非勾選習題、勾選習題類似題.
證明題過程略過.