

科目名稱: 微積分(上)(B群)

考試時間: 1月10日第二節

I. 填充題. (25分)

1. Let  $f(x) = 30x^{\frac{2}{3}} - 3x^{\frac{5}{3}}$ . Then  $f$  has a relative maximum at 4.

2. Let  $f(x) = x^2 - 2\sin(x^2)$ ,  $0 \leq x \leq \sqrt{2\pi}$ . Is  $f$  increasing or decreasing on  $(1.2, 2)$ .

Ans= increasing

3. Let  $f(x) = \frac{x^2 + 1}{4x^2 + 12x + 9}$ . Then  $f$  has a vertical asymptote  $x = \underline{-\frac{3}{2}}$

4. Find  $\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{x^2} \right) = \underline{-\infty}$

5. Find  $\lim_{x \rightarrow \infty} \left( \frac{x-1}{x^2+1} + \frac{5x\sqrt{x+3}}{3\sqrt{x^3+1}} + \frac{x+7}{\sqrt[3]{x^2+2x+7}} \right) = \underline{\infty}$

II. 計算、證明題. (80 分)

1. Determine the intervals where the function  $f(x) = x^3 - 3x + 2$  is increasing or decreasing.
2. Find the relative extrema of  $f(x) = x\sqrt{4-x}$ .
3. Let  $L(x) = ax+b$ , determine the constants  $a$  and  $b$  such that  $\lim_{x \rightarrow \infty} \left( \frac{2x^2 + 3}{x + 1} - L(x) \right) = 0$ .
4. Let  $f(x) = \frac{1}{1 - \cos x}$ ,  $0 < x < 2\pi$ . Find the relative extrema of  $f$ .
5. Determine the intervals where the graph of  $f(x) = x^4 - 4x^3 + 12$  is concave upward and the intervals where it is concave downward.
6. Find all inflection points of the function  $g(x) = x^3 - 6x^2 + 2x + 3$ .
7. Use the Second Derivative Test to find the relative extrema of  $f(x) = 2x^4 - 8x + 4$  if the relative extrema exist.
8. Let  $f(x) = \sin^2 x$ ,  $0 < x < \frac{3\pi}{2}$ . Find the relative extrema of  $f$ .
9. Find the limit. (a)  $\lim_{x \rightarrow 0^+} \cot 2x$  (b)  $\lim_{x \rightarrow -\infty} \frac{2t^2}{\sqrt{t^4 + t^2}}$ .
10. Find the horizontal and vertical asymptotes of the function  $f(x) = \frac{x}{x+1}$ .

題號	答案
1	increasing interval is $(-\infty, -1), (1, \infty)$ , decreasing interval is $(-1, 1)$
2	$f$ has relative maximum at $x = \frac{8}{3}$
3	$a = 2, b = -2$
4	$f$ has relative minimum at $x = \pi, f(\pi) = \frac{1}{2}$
5	concave upward: $(-\infty, 0), (2, \infty)$ , concave downward: $(0, 2)$
6	$g$ has an inflection point $(2, -9)$
7	relative minimum at $x = 1$ , with value $f(1) = -2$
8	relative maximum at $x = \frac{\pi}{2}$ , with value $f(\frac{\pi}{2}) = 1$ , relative minimum at $x = \pi$ , with value $f(\pi) = 0$ .
9	(a) $\infty$ (b) 2
10	$y = 1$ is a horizontal asymptote and $x = -1$ is a vertical asymptote.

\* 為非勾選習題、勾選習題類似題。  
證明題過程略過。