

科目名稱: 微積分(下)(A群)

考試時間: 4 月 17 日第二節

I. 填充題. (25 分)

1. Find $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx = \underline{\frac{\pi}{4}}$

2. Find $\int_2^{\infty} \frac{1}{x^2} \, dx = \underline{\frac{1}{2}}$

3. Find $\lim_{(x,y) \rightarrow (0,0)} \frac{x-y}{\sqrt{x}-\sqrt{y}} = \underline{\text{(送分)}}$

4. The domain of $f(x, y) = \ln(5 - x - y)$ is $\underline{x + y < 5}$

5. The range of $f(x, y) = \sqrt{4 - x^2 - y^2}$ is $\underline{0 \leq z \leq 2}$

II. 計算、證明題. (80 分)

1. (a) Find $\int \sec^3 x \tan^3 x \, dx$ (b) Find $\int \sec^4 x \, dx$.

2. (a) Find $\int \sin^3 x \cos^2 x \, dx$

(b) Find $\int \sin 5x \sin 4x \, dx$. (Hint: $\sin mx \sin nx = \frac{1}{2} [\cos(m-n)x - \cos(m+n)x]$)

3. Find $\int \frac{1}{x^2 \sqrt{9-x^2}} \, dx$.

4. Find $\int \frac{1}{(x^2+1)^{\frac{3}{2}}} \, dx$.

5. Find $\int_5^{10} \frac{\sqrt{x^2-25}}{x} \, dx$.

6. Find $\int \frac{x^2-6x+2}{x^3+2x^2+x} \, dx$.

7. Find $\int_1^2 \frac{x+1}{x(x^2+1)} \, dx$.

8. (a) Find $\int_0^1 \frac{1}{\sqrt[3]{x}} \, dx$ (b) Find $\int_0^\infty \frac{1}{\sqrt{x}(x+1)} \, dx$.

9. (a) Find $\lim_{(x,y) \rightarrow (1,2)} \frac{xy}{x^2+y^2}$. (b) Find $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$.

10. Find $\lim_{(x,y) \rightarrow (0,0)} (x^2+y^2) \ln(x^2+y^2)$. (Hint: Use polar coordinates.)

題號	答案	來源
1	(a) $\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$, (b) $\frac{1}{3} \tan^3 x + \tan x + C$	8.3 - 習題 27*, 22
2	(a) $\frac{-\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$, (b) $\frac{1}{2} \sin x - \frac{1}{18} \sin 9x + C$	8.3 - 習題 7, 48
3	$\frac{-1}{9} \frac{\sqrt{9-x^2}}{x} + C$	8.4 - 例題 1
4	$\frac{x}{\sqrt{x^2+1}} + C$	8.4 - 例題 3
5	$5\sqrt{3} - \frac{5}{3}\pi$	8.4 - 習題 8
6	$2 \ln x - \ln x+1 + \frac{9}{x+1} + C$	8.5 - 習題 13
7	$\frac{3}{2} \ln 2 - \frac{1}{2} \ln 5 + \arctan 2 - \arctan 1$	8.5 - 習題 23
8	(a) $\frac{3}{2}$ (b) π	8.8 - 例題 6, 7
9	(a) $\frac{2}{5}$ (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$ does not exist.	13.2 - 習題 17*
10	0	13.2 - 習題 60

* 為非勾選習題、勾選習題類似題。
證明題過程略過。