中原大學 112 學年度 □上學期 ■下學期 考試命題紙 ■第二次會考

科目名稱: 微積分 (下)(A 群)
考試時間: 5 月 15 日第二節

I. 填充題. (25 分)

1. Let $f(x,y,z) = x^2 \sin z + e^{2z} \ln y$. Evaluate the derivative $f_{zzy}(2,3,0) =$ $\frac{4}{3}$

2. Let $z = e^{2x}y + y^4$, where $x = \cos t, y = \sin t$. Find $\left.\frac{dz}{dt}\right|_{t=\frac{\pi}{2}} =$ -2

3. Let θ be the angle of inclination of the tangent plane to the surface $z = x^2 - xy + y^2$ at the point $(2,1,3)$. Evaluate $\cos\theta =$ $\frac{1}{\sqrt{10}}$

4. Let $f(x,y) = e^{-3x}y^4$. Let **u** be the unit vector **u** that minimizes the directional derivative $D_{\mathbf{u}}f$ at the point $(0,1)$.

   · What is the unit vector **u** = $< \frac{3}{5}, -\frac{4}{5} >$

   · What is the corresponding directional derivative $D_{\mathbf{u}}f =$ -5

II. 計算、證明題. (80 分)

1. Let $f(x,y,z) = (\sin x)^2 y^3 + 2xye^z - 3yz$. Find the derivative f_{xy} and f_{yz}.

2. Let $w = x\cos(yz)$ and $x = s^2, y = t^2, z = s - 2t$. Find the partial derivatives $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$.

3. Consider the equation $\cos(xy) + \sin(yz) + z = 20$. Find the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

4. Find the directional derivative of the function $f(x,y) = 3x^2 - y^2 + 4$ at $(1,2)$ in the direction from $P(-1,4)$ to $Q(3,6)$.

5. Find the unit vector $\mathbf{u}$ that maximizes the directional derivative $D_{\mathbf{u}}T$ of the function $T(x,y) = 80 - 3x^2 - y^2$ at $P(-1,5)$ in the directional of the unit vector $\mathbf{u}$. What is the corresponding the directional derivative $D_{\mathbf{u}}T$ at $P(-1,5)$.

6. Find the tangent plane and the normal line to the surface $z = ye^{2xy}$ at $(0,2,2)$.

7. Find a set of parametric equation for the tangent line to the curve of intersection of the surfaces $x^2 + y^2 + z^2 = 14, x - y - z = 0$ at the point $(3,1,2)$.

8. Find an equation of the tangent plane to the surface and a set of symmetric equation for the normal line to the surface $z = x^2 - y^2$ at the point $(3,2,5)$.

9. Verify that the function $z = e^{x-y}\cos(x+y)$ satisfies the Laplace's equation $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$.

10. Find the gradient of the function $f(x,y) = \dfrac{\ln(x^2+y^4)}{x} + x^3 - 3xy^2 - 4\sin(x^2y^3)$.

112 學年度第二學期理、工、電資學院微積分 (A 群) 第二次會考答案 2024.5.15

題號	答案	來源
1	$f_{xy} = 6y^2 \sin x \cos x + 2e^z, f_{yz} = 2xe^z - 3$	13.3 – 例題 7
2	$\frac{\partial w}{\partial s} = 2s\cos(st^2 - 2t^3) - s^2t^2\sin(st^2 - 2t^3),$ $\frac{\partial w}{\partial t} = -2t(s^3 - 2s^2t)\sin(st^2 - 2t^3) + 2s^2t^2\sin(st^2 - 2t^3)$	13.5 – 習題 22
3	$\frac{\partial z}{\partial x} = \frac{y\sin(xy)}{y\cos(yz)+1}, \frac{\partial z}{\partial y} = \frac{x\sin(xy)+z\cos(yz)}{y\cos(yz)+1}$	13.5 – 習題 37
4	$\frac{8\sqrt{5}}{5}$	13.6 – 習題 26
5	$\mathbf{u} = \frac{\sqrt{34}}{34}(3,-5), D_{\mathbf{u}}T(-1,5) = 2\sqrt{34}$	13.6 – 習題 57
6	The tangent plant: $8x + y - z = 0$, The normal line: $\frac{x}{8} = y - 2 = \frac{z-2}{-1}$	13.7 – 習題 27
7	$\begin{cases} x = 3 + 2t \\ y = 1 + 10t \\ z = 2 - 8t \end{cases}$	13.7 – 習題 33
8	The tangent plant: $6x - 4y - z = 5$, The normal line: $\frac{x-3}{6} = \frac{y-2}{-4} = \frac{z-5}{-1}$	13.7 – 習題 9,13,14
9	略	13.3 – 習題 97
10	$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right).$ $\frac{\partial f}{\partial x} = -\frac{\ln(x^2+y^4)}{x^2} + \frac{2}{x^2+y^4} - 3y^2 + 3x^2 - 8xy^3\cos(x^2y^3),$ $\frac{\partial f}{\partial y} = \frac{4y^3}{x(x^2+y^4)} - 6xy - 12x^2y^2\cos(x^2y^3).$	13.6 – 習題 17

* 為非勾選習題、勾選習題類似題.
證明題過程略過.