

科目名稱: 微積分(下)(A群)

考試時間: 6 月 19 日第二節

I. 填充題. (25 分)

1. Change the order of the integration $\int_1^{10} \int_0^{\ln y} f(x, y) dx dy = \underline{\int_0^{\ln 10} \int_{e^x}^{10} f(x, y) dy dx}$

2. Evaluate $\iint_R \left(1 - \frac{1}{2}x^2 - \frac{1}{2}y^2\right) dA$, where R is the region given by $0 \leq x \leq 1, 0 \leq y \leq 1$.

Ans = $\underline{\frac{2}{3}}$

3. Rewrite the region $R = \{(x, y) | 0 \leq y \leq \sqrt{4 - x^2}, -2 \leq x \leq 2\}$ by using the polar coordinates

$R_{new} = \{(r, \theta) | \underline{0 \leq r \leq 2, 0 \leq \theta \leq \pi}\}$

4. Use the binomial series to express $\frac{1}{(1+x)^2}$ as a power series S . Ans: $S = \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)x^n}{1}$

5. Find all critical points of the function $f(x, y) = \sqrt{49 - (x-2)^2 - y^2}$. Ans = $\underline{(2, 0)}$

II. 計算、證明題. (80 分)

- (a) Find the 3rd Taylor polynomial $P_3(x)$ of the function $f(x) = \ln x$ centered at 1.
(b) Use the above result to approximate the value of $\ln(1.3)$.
- Find the absolute extrema of the function $f(x, y) = x^2 - 4xy + 5$ over the region $R = \{(x, y) : 1 \leq x \leq 4, 0 \leq y \leq 2\}$.
- Find the relative extrema of $f(x, y) = -x^3 + 4xy - 2y^2 + 1$.
- Use an iterated integral to find the area of the region bounded by the graphs of $2x - 3y = 0$, $x + y = 5$ and $y = 0$.

5. Evaluate $\int_0^1 \int_{2x}^2 4e^{y^2} dy dx$.

- Find the volume of the solid bounded by the graphs of $z = x + y$, $x^2 + y^2 = 4$ and the first octant. (Note: first octant: $\{(x, y, z) | x \geq 0, y \geq 0, z \geq 0\}$)
- Find the average value of $f(x, y) = e^{x+y}$ over the triangle with vertices $(0, 0)$, $(0, 1)$ and $(1, 1)$.
- Find the volume of the solid region bounded by the graphs $z = \ln(x^2 + y^2)$, $z = 0$, $x^2 + y^2 \geq 1$, $x^2 + y^2 \leq 4$. (Hint: Use the polar coordinates)

9. Evaluate the integral by converting to the polar coordinates:

(a) $\int_0^3 \int_0^{\sqrt{9-y^2}} y dx dy$ (b) $\iint_R e^{-\frac{x^2+y^2}{2}} dA, R : x^2 + y^2 \leq 25, x \geq 0$.

10. (a) Find the 4th Maclaurin polynomial $P_4(x)$ of the function $f(x) = e^{-x^2}$.

(b) Approximate the integral $\int_0^1 e^{-x^2} dx$ using the above result. (i.e., $e^{-x^2} \approx P_4(x)$)

112 學年度第二學期理、工、電資學院微積分 (A 群) 期末考答案 2024.6.19

題號	答案	來源
1	(a) $P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$, (b) 0.264	講義
2	$f(4, 0) = 21$ is absolute max. and $f(4, 2) = -11$ is absolute min.	13.8 – 習題 39
3	$f\left(\frac{4}{3}, \frac{4}{3}\right)$ has a relative maximum	13.8 – 例題 3
4	Area = 5	14.1 – 習題 38
5	$e^4 - 1$	14.1 – 習題 63
6	$\frac{16}{3}$	14.2 – 習題 31
7	$1 - 2e + e^2$	14.2 – 習題 55
8	$8\pi \ln 2 - 3\pi$	14.3 – 習題 36
9	(a) 9, (b) $(1 - e^{-25/2})\pi$	14.3 – 習題 23
10	(a) $P_4(x) = 1 - x^2 + \frac{1}{2}x^4$, (b) $1 - \frac{1}{3} + \frac{1}{10} = \frac{23}{30}$	講義

* 為非勾選習題、勾選習題類似題。
證明題過程略過。