

科目名稱: 微積分(下)(B群)

考試時間: 6 月 19 日第二節

I. 填充題. (25 分)

1. The third term of the Maclaurin series representation for $f(x) = \tan x$ is $\frac{2}{15}x^5$

2. The domain of $g(x, y) = \frac{\ln(x+y+1)}{y-x}$ is $\{(x, y) : y \neq x, x+y+1 > 0, x, y \in R\}$

3. If $f(x, y) = e^{x^2y} + xy^2 + x + y^2$, then $\frac{\partial f}{\partial x} = \frac{(2xy)e^{x^2y} + y^2 + 1}{1}$

4. If $f(x, y) = x^2y + e^{xy} + 1$, then $f_{xy} = \frac{2x + e^{xy} + xye^{xy}}{1}$

5. If $f(x, y) = \int_x^{x+y} \cos t \, dt$, then $\frac{\partial f}{\partial x} = \frac{\cos(x+y) - \cos x}{1}$

II. 計算、證明題. (80 分)

1. Find the sum of the series $\sum_{n=1}^{\infty} nx^n, |x| < 1$.
2. Find the Taylor polynomial $P_3(x)$ of degree 3 for $f(x) = \sqrt{x}$ at $c = 1$.
3. Find the Maclaurin polynomial $P_2(x)$ for $f(x) = e^x \sin x$.
4. Find and sketch the domain of $f(x, y) = \sqrt{y^2 - x}$.
5. Sketch the graph of $f(x, y) = x^2 + 4y^2$. (只要畫出圖形)
6. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y}{x^2 + y^2}$ exist.
7. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ does not exist.
8. Let $f(x, y) = 4e^{xy-1} - 2x^2 - y^2$. Find the slope of the tangent line at the point $(1, 1, 1)$ on the curve formed by the intersection of the surface $z = f(x, y)$ and the plane $y = 1$.
9. Verify that $f_{xy} = f_{yx}$ if $f(x, y) = x \cos xy^2$.
10. Suppose z is a differentiable function of x and y that is defined implicitly by $x^2 + y^3 + z + 2yz^2 = 5$.
Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

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題號	答案	來源
1	$\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2}$	8.7 - 習題 20(a)
2	$P_3(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$	13.5 - 例題 2(a)*
3	$P_2(x) = x + x^2$	13.5 - 習題 6
4	$D : \{(x, y) \in \mathbb{R}^2 : y^2 - x \geq 0\}$	13.6 - 例題 3
5	略	13.6 - 例題 5
6	略	13.7 - 例題 4*
7	略	13.7 - 例題 2
8	$\frac{\partial f}{\partial x}(1, 1) = 0$	13.7 - 例題 3*
9	略	13.3 - 例題 2*
10	$\frac{\partial z}{\partial x} = -\frac{2x}{1+4yz}, \quad \frac{\partial z}{\partial y} = -\frac{3y^2 + 2z^2}{1+4yz}$	13.6 - 例題 7*

* 為非勾選習題、勾選習題類似題。
證明題過程略過。