科目名稱: 微積分 (上)(B 群)
考試時間: 10 月 9 日第二節

I. 填充題. (25 分)

1. Find $\lim\limits_{x \to 2^-} \dfrac{1}{x^3 - 8} = $ <u>$-\infty$ or 不存在</u>

2. Find $\lim\limits_{h \to 0} \dfrac{(1 + h)^{100} - 1}{h} = $ <u>100</u>

3. Let $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & , x \neq 2 \\ a & , x = 2 \end{cases}$, find the value of a that makes f continuous everywhere,
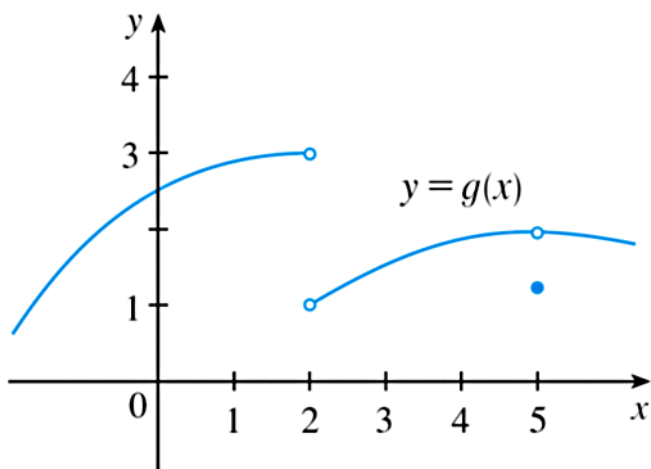
$a = $ <u>3</u>

4. If $f(x) = x^3 - x$, $f''(1) = $ <u>6</u>

5. Let $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, $g'(5) = 2$, find the value of $(f + 4g)'(5) = $ <u>14</u>

II. 計算、證明題. (80 分)

1. Find (a) $\lim\limits_{x \to 2^-} g(x)$ (b) $\lim\limits_{x \to 2^+} g(x)$ (c) $\lim\limits_{x \to 2} g(x)$.

2. Find the vertical asymptote of the function $f(x) = \dfrac{2x}{x - 3}$.

3. Find (a) $\lim\limits_{x \to 3} \dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3}$ (b) $\lim\limits_{h \to 0} \dfrac{\sqrt{9 + h} - 3}{h}$.

4. Show that the function $f(x) = \begin{cases} x^4 \sin\left(\dfrac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is continuous at 0.

5. Show that there is a solution of the equation $x^3 + x^2 - x - 5 = 0$ between 0 and 2.

6. Show that (a) $f(x) = |x|$ is continuous at $x = 0$.
   (b) $f(x) = |x|$ is not differentiable at $x = 0$.

7. Find the derivatives of the function $f(x)$ using the definition of the derivative.
   (a) $f(x) = x^2$ (b) $f(x) = \sqrt{x}$.

8. Let $h(x) = x^2 g(x)$, and $f(x) = \dfrac{g(x)}{x}$, if $g(3) = 5$, $g'(3) = 2$. Find (a) $h'(3)$ (b) $f'(3)$

9. Let $f(x) = \dfrac{\sqrt{x}}{x - 3}$ and $g(x) = (3x^2 - 5x)(2x + 3)$. Find $f'(x)$ and $g'(x)$.

10. If $\lim\limits_{x \to 1} \dfrac{f(x)}{(x - 1)^2} = 5$, find (a) $\lim\limits_{x \to 1} f(x)$ (b) $\lim\limits_{x \to 1} \dfrac{f(x)}{x - 1}$.

題號	答案
1	$(a)3,$ $(b)1,$ $(c) \lim\limits_{x \to 2} g(x)$ does not exist.
2	$x = 3$ is a vertical asymptote
3	$(a) -\dfrac{1}{9},$ $(b)\dfrac{1}{6}$
4	略
5	略
6	略
7	$(a)f'(x) = 2x,$ $(b)f'(x) = \dfrac{1}{2\sqrt{x}}$ (* 題目有限定解法，用錯方法即不給分)
8	$(a)h'(3) = 48,$ $(b)f'(3) = \dfrac{1}{9}$
9	$f'(x) = \dfrac{\frac{1}{2}x^{-1/2}(x-3) - x^{1/2}}{(x-3)^2}, g'(x) = 18x^2 - 2x - 15$
10	$(a)\lim\limits_{x \to 1} f(x) = 0,$ $(b)\lim\limits_{x \to 1} \dfrac{f(x)}{x-1} = 0$

* 為非勾選習題、勾選習題類似題.
證明題過程略過.