

科目名稱: 微積分(上)(A群)

考試時間: 11 月 6 日第二節

I. 填充題. (25 分)

1. If $f(x) = \frac{\tan x}{1 + \sec x}$ and $f'(a) = 0, 0 \leq a \leq 2\pi$, then $a =$ 送分

2. Find $\frac{d^{101}}{dx^{101}}(\cos x) =$ $-\sin x$

3. If $y = \cos(\sin(\tan x))$, then $y' =$ $-\sin(\sin(\tan x)) \cos(\tan x) \sec^2 x$

4. If $f(x) + x^2 \cos f(x) = x$, then $f'(0) =$ 1

5. If $y = \theta^3 \sec \theta$, then $dy =$ $[3\theta^2 \sec \theta + \theta^3 \sec \theta \tan \theta] d\theta$ *沒寫 $d\theta$ 或寫成 dx 不給分*

II. 計算、證明題. (80 分)

- Find (a) $\lim_{x \rightarrow 0} \frac{\tan 2x}{x}$ (b) $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x^3 - 4x}$.
- Differentiate (a) $y = (x^3 - 1)^{100}$ (b) $y = \sin^2 \left(\frac{1}{x} \right)$.
- If $\tan \left(\frac{y}{x} \right) = x + y$, find y' .
- Find an equation of the tangent line to the curve $x^2 - xy - y^2 = 1$ at $(2, 1)$.
- (a) If $F(x) = f(g(x))$, where $f(5) = 2$, $f'(2) = 1$, $f'(5) = 2$, $g(3) = 5$, $g'(3) = 2$. Find $F'(3)$.
(b) If $h(x) = \sqrt{2 + 2f(x)}$, where $f(1) = 7$ and $f'(1) = 2$. Find $h'(1)$.
- If $y = \tan x$, (a) find the differential dy and (b) evaluate dy given $x = \frac{\pi}{4}$ and $dx = -0.1$.
- Use a linear approximation (or differential) to estimate $\sqrt{99.5}$.
- Let $y = x \sin x$. Find (a) $\frac{dy}{dx}$, (b) $\frac{d^2y}{dx^2}$, (c) $\frac{d^4y}{dx^4}$ and (d) $\frac{d^{35}y}{dx^{35}}$.
- Find the critical numbers of $f(x)$: (a) $f(x) = |3 - 2x|$ (b) $f(x) = x^{4/5}(x - 4)^2$.
- Find the absolute maximum and absolute minimum values of $f(t) = (t^2 - 4)^3$ on $[-2, 3]$.

題號	答案	來源
1	(a) 2, (b) $-\frac{3}{4}$	2.5 - 習題 51, 53*
2	(a) $100(x^3 - 1)^{99} \cdot 3x^2$, (b) $2 \sin\left(\frac{1}{x}\right) \cos\left(\frac{1}{x}\right) \cdot \frac{-1}{x^2}$	2.5 - 例題 3, 習題 34*
3	$y' = \frac{x^2 + y \sec^2\left(\frac{y}{x}\right)}{x \sec^2\left(\frac{y}{x}\right) - x^2}$	2.6 - 習題 15*
4	$y - 1 = \frac{3}{4}(x - 2)$	2.6 - 習題 29
5	(a) $F'(3) = 4$, (b) $h'(1) = \frac{1}{2}$	2.5 - 習題 63*, 64*
6	(a) $dy = \sec^2 x dx$, (b) $dy = -0.2$	2.9 - 習題 19
7	9.975	2.9 - 習題 34*
8	(a) $\sin x + x \cos x$, (b) $2 \cos x - x \sin x$, (c) $-4 \cos x + x \sin x$, (d) $-35 \sin x - x \cos x$	2.4 - 習題 62*
9	(a) $x = \frac{3}{2}$ is a critical number, (b) $f'(x) = 0 \Rightarrow x = 4$ and $x = \frac{8}{7}$ are critical numbers, $f'(x)$ 不存在 $\Rightarrow x = 0$ is a critical number.	3.1 - 習題 41
10	$f(0) = -64$ is absolute min. , $f(3) = 125$ is absolute max.	3.1 - 習題 54

* 為非勾選習題、勾選習題類似題。
證明題過程略過。