

科目名稱: 微積分(上)(B群)

考試時間: 11 月 6 日第二節

I. 填充題. (25 分)

1. $\lim_{h \rightarrow 0} \frac{(2+h)^5 - 32}{2h} = \underline{40}$

2. Let $y = 2x + b$ be an equation of the tangent line to $y = x^2 + 3$ at $x = 1$. Then $b = \underline{2}$

3. Let $h(x) = xf(x)$. If $f(1) = 2$ and $f'(1) = 1$, then $h'(1) = \underline{3}$

4. Let $f(x) = 2\pi$. Then $f'(2) = \underline{0}$

5. Let $f(x) = |x|$. Then f is not differentiable at $x = \underline{0}$

II. 計算、證明題. (80 分)

1. Use the position function $s(t) = \sqrt{t + 16}$ meters (公尺) to find the velocity (速度) $v(t)$ at time $t = 2$ seconds (秒).

2. Use the definition $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ to find $f'(1)$ for $f(x) = 3x + 1$.

3. Compute the left-hand derivative $D_-f(0)$ for $f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & x \geq 0 \end{cases}$.

(Hint: The left-hand derivative $D_-f(0) = \lim_{h \rightarrow 0^-} \frac{f(h) - f(0)}{h}$)

4. If f is differentiable at $x = 2$, evaluate $\lim_{x \rightarrow 2} \frac{[f(x)]^2 - [f(2)]^2}{x^2 - 4}$.

5. Find $f'(x)$ for $f(x) = (\sqrt{x} + 3x) \left(5x^2 - \frac{3}{x}\right)$.

6. Find $h'(t)$ for $h(t) = t(\sqrt[3]{t} + 3)$.

7. Find $f'(x)$ for $f(x) = (x^2 - 1) \left(\frac{x^3 + 3x^2}{x^2 + 2}\right)$.

8. Find $f'(x)$ for $f(x) = x^{\frac{2}{3}}(x^2 - 2)(x^3 - x + 1)$.

9. Let $f(x) = 3x^3 + 2x + 5$ and $g(x) = x^{\frac{3}{2}} + 2$. Find $h'(0)$ for $h(x) = \frac{f(x)}{g(x)}$.

10. Let $f(0) = 3$, $f'(0) = 2$, $g(0) = -1$, $g'(0) = 1$. Find $h'(0)$ for $h(x) = f(x)g(x) + \frac{2}{g(x)}$.

113 學年度第 1 學期理、工、電資學院微積分 (B 群) 期中考答案 2024.11.6

題號	答案	來源
1	$\frac{1}{2\sqrt{18}}$	2.1 - 習題 13(b)
2	$f'(1) = 3$	2.2 - 例題 1*
3	$D_x f(0) = 0$	2.2 - 習題 15
4	$\frac{f'(2)f(2)}{2}$	2.2 - 習題 23
5	$f'(x) = \frac{25}{2}x^{3/2} + \frac{3}{2}x^{-3/2} + 45x^2$	2.4 - 習題 3
6	$h'(t) = \frac{4}{3}t^{1/3} + 3$	2.4 - 習題 13
7	$f'(x) = \frac{(5x^4 + 12x^3 - 3x^2 - 6x)(x^2 + 2) - (x^5 + 3x^4 - x^3 - 3x^2) \cdot 2x}{(x + 2)^2}$ (* 除法公式正負號錯誤即不給分)	2.4 - 習題 15
8	$f'(x) = (\frac{8}{3}x^{5/3} - \frac{4}{3}x^{-1/3})(x^3 - x + 1) + (x^{8/3} - 2x^{2/3})(3x^2 - 1)$	2.4 - 習題 33
9	$h'(0) = 1$	2.4 - 定理 4 應用
10	$h'(0) = 1$	2.4 - 習題 21*, 24*

* 為非勾選習題、勾選習題類似題。
證明題過程略過。