

科目名稱: 微積分 (上)(A 群)

考試時間: 12 月 4 日 第二節

I. 填充題. (25 分)

1. Suppose $f(1) = 2$ and $f'(x) \geq 3$ for all real numbers x , how small can $f(4)$ possibly be? 11

2. The local minimum of $f(x) = x^{5/3} - 5x^{2/3}$ occurs at $x =$ 2

3. $\lim_{x \rightarrow -\infty} \frac{\sqrt{1+2x^6}}{3-x^3} =$ $\sqrt{2}$

4. The horizontal asymptote of the curve $y = x^2 \sin \frac{1}{x^2}$ is $y = k$, where $k =$ 1

5. The slant asymptote of the curve $y = \frac{x^2+4}{x+1}$ is $y = x + c$, where $c =$ -1

II. 計算、證明題. (80 分)

- Find the number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \sqrt[3]{x}$ on the closed interval $[0, 1]$.
- Show that the equation $x^4 + 4x - 1 = 0$ (a) has at least two real solutions, and (b) has at most two real solutions using Rolle's Theorem.
- Find the local maximum and minimum values of $f(x) = 1 + 3x^2 - 2x^3$, by using (a) the First Derivative Test, (b) the Second Derivative Test.
- (a) Find the open intervals on which $f(x) = \frac{x^2 - 24}{x - 5}$ is increasing or decreasing.
(b) Find the local maximum and minimum values of the above function.
- (a) Find the open intervals on which $f(x) = \sin^2 x - \cos 2x$, $0 < x < \pi$, is concave upward or concave downward. (b) Find the inflect points of the above function. (Hint: $\sin^2 x = \frac{1 - \cos 2x}{2}$)
- (a) Find $\lim_{x \rightarrow \infty} (\sqrt{x^2 + ax} - \sqrt{x^2 + bx})$ (b) Find $\lim_{x \rightarrow \infty} \frac{\sin^2 x}{x^2 + 1}$.
- Use the Mean Value Theorem to prove that $|\sin a - \sin b| \leq |a - b|$ for all real numbers a and b .
- Sketch the graph of a function that satisfies all of the following conditions $f(1) = f'(1) = 0$,
 $\lim_{x \rightarrow 2^+} f(x) = \infty$, $\lim_{x \rightarrow 2^-} f(x) = -\infty$, $\lim_{x \rightarrow 0} f(x) = -\infty$, $\lim_{x \rightarrow -\infty} f(x) = \infty$,
 $\lim_{x \rightarrow \infty} f(x) = 0$, $f''(x) > 0$ for $x > 2$, $f''(x) < 0$ for $x < 0$ and for $0 < x < 2$.
- Sketch the graph of the function $f(x) = x^{2/3}(6 - x)^{1/3}$.
(Hint: $f'(x) = \frac{4 - x}{x^{1/3}(6 - x)^{2/3}}$, $f''(x) = \frac{-8}{x^{4/3}(6 - x)^{5/3}}$)
- Sketch the graph of the curve $y = \frac{x^3 + 4}{x^2}$ by considering increasing/decreasing intervals, concavity, and vertical and slant asymptotes.

題號	答案	來源
1	$c = \frac{1}{\sqrt{27}}$	3.2 – 習題 17
2	證明題	3.2 – 習題 26*
3	(a) The local maximum is : $f(1) = 2$ and local minimum is : $f(0) = 1$ (b) The local maximum is : $f(1) = 2$ and local minimum is : $f(0) = 1$	3.3 – 習題 23
4	(a) Increasing : $(-\infty, 4), (6, \infty)$, Decreasing : $(4, 5), (5, 6)$ (b) The local minimum is : $f(4) = 8$, and local maximum is : $f(6) = 12$	3.3 – 習題 13
5	(a) concave upward : $(0, \frac{\pi}{4}), (\frac{3\pi}{4}, \pi)$, concave downward : $(\frac{\pi}{4}, \frac{3\pi}{4})$ (b) Inflection points : $(\frac{\pi}{4}, f(\frac{\pi}{4})) = (\frac{\pi}{4}, \frac{1}{2}), (\frac{3\pi}{4}, f(\frac{3\pi}{4})) = (\frac{3\pi}{4}, \frac{1}{2})$	3.3 – 習題 17
6	(a) $\frac{1}{2}(a - b)$, (b) 0	3.4 – 習題 30
7	證明題	3.2 – 習題 35
8	做圖題	3.4 – 習題 61
9	做圖題	3.3 – 例題 7
10	做圖題	3.5 – 習題 55

* 為非勾選習題、勾選習題類似題。
證明題、做圖題過程略過。