中原大學 113 學年度 ■上學期 □下學期 考試命題紙 ■期末考

科目名稱: 微積分 (上)(A 群)
考試時間: 1 月 8 日第二節

I. 填充題. (25 分)

1. Find the interval $[a, b] =$ $\underline{[-1, 2]}$ for which the value of the integral $\int_a^b (-x^2 + x + 2)\, dx$ is a maximum.

2. Find $\frac{d}{dx} \int_1^x \sqrt{t + t^3}\, dt =$ $\underline{\sqrt{x + x^3}}$

3. If $f(x)$ is continuous function on $[0, 1]$ and $\int_0^1 f(x)\, dx = \lim_{n\to\infty} n^2 \sum_{i=1}^{n} \frac{i}{n^4 + i^4}$ (Riemann Sum of $f(x)$ on $[0, 1]$), then $f(x) =$ $\underline{\frac{x}{1 + x^4}}$

4. If $f(x) = 3x + \cos x$, then $(f^{-1})'(1) =$ $\underline{\frac{1}{3}}$

5. Find $\cos\left(\sin^{-1}\left(\frac{5}{13}\right)\right) =$ $\underline{\frac{12}{13}}$

II. 計算、證明題. (80 分)

1. (a) If $\int_0^9 f(x)\,dx = 5$ and $\int_0^9 g(x)\,dx = 4$, find $\int_0^9 (2f(x) + 3g(x))\,dx$.

   (b) Find $\int_0^{\pi} \frac{d}{dx}\left(\sqrt{\sin\frac{x}{2}}\,\cos\frac{x}{3}\right)\,dx$.

2. Find $\int_0^1 \left(2|2x-1| + 3\sqrt{1-x^2}\right)\,dx$.

3. (a) Find $\frac{d}{dx}\int_x^{x^4} \sec t\,dt$. (b) Let $\int_0^{x^2} f(t)\,dt = x\sin\pi x$. Find $f(4)$.

4. (a) Find $\int \frac{1+\sqrt{t}+t+t^2}{\sqrt{t}}\,dt$. (b) Find $\int (2t+1)^{100}\,dt$.

5. (a) Find $\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}}\,dx$. (b) Find $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{\tan x}{1+x^2+x^4} + 1\right)\,dx$.

6. (a) Find $\int_1^4 \left(t^2 - t - 6\right)\,dt$. (b) Find $\int_1^4 \left|t^2 - t - 6\right|\,dt$.

7. Find $\int \sqrt{1+x^2}\cdot x^3\,dx$.

8. Let $f(x) = \int_2^x \sqrt{1+t^4}\,dt$. Find $(f^{-1})'(0)$.

9. (a) Let $f(x) = \sin^{-1}(2x)$. Find $f'(0)$. (b) Let $f(x) = x\arctan(\sqrt{x})$. Find $f'(x)$.

10. (a) Find $\int_0^{\frac{1}{4}} \frac{1}{1+16x^2}\,dx$. (b) Find $\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}}\,dx$.

113 學年度第 1 學期理、工、電資學院微積分 (A 群) 期末考答案 2025.1.8

題號	答案	來源
1	$(a)\ 22, \quad (b)\ \frac{1}{2}$	$(a)4.2-$ 習題 59, $(b)4.4-$ REVIEW*
2	$1+\frac{3}{4}\pi$	$4.2-$ 習題 44*
3	$(a)\ -\sec x+4x^3\sec x^4, \ \ (b)\ f(4)=\frac{\pi}{2}$	$4.3-$ 例題 4*
4	$(a)\ 2t^{\frac{1}{2}}+t+\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+C, \ \ (b)\ \frac{1}{202}(2t+1)^{101}+C$	$(a)4.4-$ 習題 15, $(b)4.5-$ 習題 38*
5	$(a)\ \frac{32}{3}-4\sqrt{3}, \quad (b)\ \frac{\pi}{2}$	$4.5-$ 習題 42, 例題 9*
6	$(a)\ -\frac{9}{2}, \quad (b)\ \frac{61}{6}$	$4.4-$ 例題 6
7	$\frac{1}{5}(1+x)^{5/2}-\frac{1}{3}(1+x^2)^{3/2}+C$	$4.5-$ 例題 5*
8	$(f^{-1})'(0)=\frac{1}{\sqrt{17}}$	$6.1-$ 習題 47*
9	$(a)\ f'(0)=2, \quad (b)\ f'(x)=\arctan(\sqrt{x})+\frac{\sqrt{x}}{2(1+x)}$	$6.6-$ 習題 23, 例題 5*
10	$(a)\ \frac{\pi}{16}, \quad (b)\ \frac{\pi}{2}$	$6.6-$ 習題 64,62*

* 為非勾選習題、勾選習題類似題.
證明題、做圖題過程略過.