

科目名稱: 微積分 (上)(B 群)

考試時間: 1 月 8 日第二節

I. 填充題. (25 分)

1. Let $f(x) = \frac{1}{3}x^3 - 4x^2 + 15$, find critical numbers 0, 8

2. Let $f(x) = x^3 + 6x^2 - 2x + 6$, find the inflection point $(-2, 26)$

3. Let $f(x) = \sin x - \cos x$ and $0 \leq x \leq 2\pi$.

(a) The absolute maximum of $f(x)$ on $[0, 2\pi]$ is $\sqrt{2}$

(b) The absolute minimum of $f(x)$ on $[0, 2\pi]$ is $-\sqrt{2}$

4. Let $f(x) = x + \frac{4}{x}$. The local maximum of $f(x)$ is -4 or $f(-2)$

II. 計算、證明題. (80 分)

1. Find the absolute extrema of $f(x) = 2x^3 - 3x^2 - 12x + 5$ on the interval $[-2, 4]$.
2. Let $f(x) = x^3 - 3x + 2$.
 - (a) Find the open interval on which $f(x)$ is increasing or decreasing.
 - (b) Find the local maximum or minimum of the above function.
3. Let $f(x) = 2x^3 + 9x^2 - 24x - 10$.
 - (a) Find the open interval on which $f(x)$ is concave up or concave down.
 - (b) Find the inflection point of the above function.
4. Find the local extrema of $f(x) = x^{\frac{5}{3}} - 3x^{\frac{2}{3}}$.
5. Find absolute extrema of $f(x) = \sin^2 x - \cos 2x$ on $[0, \pi]$.
6. Find the critical numbers and local extrema of $f(x) = (3x + 1)^{\frac{2}{3}}$.
7. Let $y = \frac{x}{1 + x^3}$. Find all critical numbers and use the First Derivative test to determine all local extrema.
8. Let $f(x) = x^3 - 9x^2 + 24x - 18$. Find all critical numbers and use the Second Derivative test to determine all local extrema.
9. Let $f(x) = \frac{x^2}{x^2 - 4x + 3}$. Find all asymptotes for function $f(x)$.
10. Let $f(x) = x^4 + 4x^3 - 2$. Find all critical numbers and use the First Derivative Test to determine all local extrema.

題號	答案	來源
1	the absolute max. is $f(4) = 37$, absolute min. is $f(2) = -15$	3.2 – 例題 11
2	(a) f is increasing on $(-\infty, -1), (1, \infty)$, decreasing on $(-1, 1)$ (b) the local maximum is $f(-1) = 4$, local minimum is $f(1) = 0$	3.3 – 習題 1
3	(a) $f(x)$ is concave up on $\left(-\frac{3}{2}, \infty\right)$, concave down on $\left(-\infty, -\frac{3}{2}\right)$ (b) inflection point is $\left(-\frac{3}{2}, f\left(-\frac{3}{2}\right)\right)$	3.4 – 例題 1
4	the local max. is $f(0) = 0$, local min. is $f\left(\frac{6}{5}\right) = \left(\frac{6}{5}\right)^{5/3} - 3\left(\frac{6}{5}\right)^{2/3}$	3.3 – 例題 3
5	the absolute max. is $f(\pi/2) = 2$, absolute min. are $f(0) = -1$ and $f(\pi) = -1$	3.2 – 習題 7*
6	the critical number is $-\frac{1}{3}$, local minimum is $f(-\frac{1}{3}) = 0$	3.2 – 例題 7
7	the critical number is $\frac{1}{\sqrt[3]{2}}$, local maximum is $f\left(\frac{1}{\sqrt[3]{2}}\right) = \frac{2}{3\sqrt[3]{2}}$	3.3 – 習題 7
8	the critical numbers are 2, 4, local max. is $f(2) = 2$, local min. is $f(4) = -2$	3.4 – 習題 5*
9	Vertical asymptote are $x = 3$ and $x = 1$, Horizontal asymptote is $y = 1$	3.3 – 習題 10
10	the critical number are 0, -3, local minimum is $f(-3) = -29$	3.3 – 習題 5

* 為非勾選習題、勾選習題類似題.

證明題、做圖題過程略過.