

科目名稱: 微積分 (下)(A 群) 開課班級: A 群微積分 考試時間: 3 月 12 日第二節

I. 填充題. (25 分)

1. Find $\int_e^{e^2} \frac{\ln x}{x} dx = \underline{\frac{3}{2}}$

2. Find $\frac{d}{dx} e^x(\cos x + \sin x) = \underline{2e^x \cos x}$

3. Find $\int_0^2 10^x dx = \underline{\frac{99}{\ln 10}}$

4. Find $\lim_{x \rightarrow \infty} \left(1 + \frac{e}{x}\right)^x = \underline{e^e}$

5. Find $\int_1^e t \ln t dt = \underline{\frac{1}{4}(e^2 + 1)}$

II. 計算、證明題. (80 分)

1. (a) Let $f(x) = e^{2x} + e^{-2x}$. Find $f'(0)$. (b) Let $f(x) = e^{\sin x} + \sin(e^x)$. Find $f'(x)$.

2. (a) Find $\int (e^x + e^{-x})^2 dx$. (b) Find $\int_{\ln 2}^{\ln 3} e^x \sqrt{1 + e^x} dx$.

3. (a) Let $f(x) = 5^x + \log_5 x$. Find $f'(5)$. (b) Let $f(x) = x^{\sin x}$ for $x > 0$. Find $f'(x)$.

4. (a) Find $\int x \cdot 3^{x^2} dx$. (b) Find $\frac{d}{dx} \log_{10}(2 + \log_3(\sin x))$.

5. (a) Find $\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x^2}$. (b) Find $\lim_{x \rightarrow 0^+} x^{2x}$.

6. (a) Find $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^2}$. (b) Find $\lim_{x \rightarrow (\frac{\pi}{2})^-} (\tan x - \sec x)$.

7. Find $\int_0^1 \tan^{-1} x dx$.

8. Use logarithmic differentiation to find the derivative of $y = \frac{(x+1)^4(x-5)^3}{(x-3)^8}$.

9. (a) Let $f(x) = \ln(1 + x^4)$. Find $f'(1)$. (b) Find $\int \frac{\cos(\ln t)}{t} dt$.

10. Find $\int e^{2x} \cos x dx$.

題號	答案	來源
1	(a) 0, (b) $e^{\sin x} \cos x + \cos(e^x)e^x$	6.3* – 習題 59*, 46*
2	(a) $\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + C$, (b) $\frac{2}{3}(8 - 3\sqrt{3})$	6.3* – 習題 85, 83*
3	(a) $5^5 \ln 5 + \frac{1}{5 \ln 5}$, (b) $x^{\sin x} [\cos x \ln x + \frac{1}{x} \sin x]$	6.4* – 習題 (a) 自出, (b) 36*
4	(a) $\frac{1}{2} \frac{1}{\ln 3} 3^{x^2} + C$, (b) $\frac{1}{\ln 10} \frac{1}{2 + \log_3 \sin x} \frac{1}{\ln 3} \frac{1}{\sin x} \cos x$	6.4* – 習題 (a) 48*, (b) 自出
5	(a) 0, (b) 1	6.8 – 例題 2*, 11
6	(a) 0, (b) 0	6.8 – 例題 4*, 8
7	$\frac{\pi}{4} - \frac{1}{2} \ln 2$	7.1 – 例題 5
8	$\frac{(x+1)^4(x-5)^3}{(x-8)^8} \left[\frac{4}{x+1} + \frac{3}{x-5} - \frac{8}{x-3} \right]$	6.2* – 習題 64
9	(a) 2, (b) $\sin(\ln t) + C$	6.2* – 習題 58*, 76
10	$\frac{1}{5}e^{2x} \sin x + \frac{2}{5}e^{2x} \cos x + C$	7.1 – 例題 4

證明題、做圖題過程略過, * 為非勾選習題、勾選習題類似題.