

科目名稱: 微積分 (下)(A 群)

開課班級: A 群微積分

考試時間: 4 月 16 日 第二節

I. 填充題. (25 分)

1. Find $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx = \underline{\frac{\pi}{4}}$

2. Find $\int_0^1 x\sqrt{1-x^2} \, dx = \underline{\frac{1}{3}}$

3. Find $\int \frac{1}{4-x^2} \, dx = \lambda \ln \left| \frac{2+x}{2-x} \right| + C$, where $\lambda = \underline{\frac{1}{4}}$

4. Find $\int_1^{\infty} x^{-3} \, dx = \underline{\frac{1}{2}}$

5. If $g(x, y) = x \sin y + y \sin x$, then $g(\pi, 0) = \underline{0}$

II. 計算、證明題. (80 分)

1. Evaluate the integral (a) $\int \sin 8x \cos 5x \, dx$. (b) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}} \, dx$.

2. Evaluate the integral (a) $\int \tan^2 \theta \sec^4 \theta \, d\theta$. (b) $\int \sin x \sec^5 x \, dx$.

3. Evaluate the integral $\int \sqrt{x^2 + 1} \, dx$.

4. Evaluate the integral $\int \frac{1}{x^2 \sqrt{x^2 + 1}} \, dx$.

5. Evaluate the integral $\int \frac{5}{x(x^2 + 2x + 5)} \, dx$.

6. Evaluate the integral $\int \frac{x^2 + x + 1}{(x + 1)^2(x + 2)} \, dx$.

7. Evaluate the integral $\int \frac{\sqrt{x + 1}}{x} \, dx$.

8. Evaluate the integral (a) $\int_{-\infty}^{\infty} x e^{-x^2} \, dx$. (b) $\int_0^3 \frac{1}{x - 1} \, dx$.

9. Sketch the level curves of the function $h(x, y) = 4x^2 + y^2 + 1$ for the values $k = 1, 5, 17$.

10. Evaluate the integral $\int \frac{\sqrt{x^2 - 1}}{x^4} \, dx$.

題號	答案	來源
1	$(a) -\frac{1}{6}\cos 3x - \frac{1}{26}\cos 13x + C, (b) 2$	7.2 – 習題 45, 35*
2	$(a) \frac{1}{3}\tan^3 \theta + \frac{1}{5}\tan^5 \theta + C, (b) \frac{1}{4\cos^4 x} + C$	7.2 – 習題 22, 15
3	$\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln \sqrt{1+x^2}+x + C$	7.3 – 習題 28
4	$-\frac{\sqrt{1+x^2}}{x} + C$	7.3 – 習題 14*
5	$\ln x - \frac{1}{2}\ln x^2+2x+5 - \frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C$	7.4 – 習題
6	$-2\ln x+1 - \frac{1}{x+1} + 3\ln x+2 + C$	7.4 – 習題 19
7	$2\sqrt{x+1} - \ln \sqrt{x+1}+1 + \ln \sqrt{x+1}-1 + C$	7.4 – 習題 50*
8	$(a) 0, (b) \int_0^3 \frac{1}{x-1} dx \text{ is diverges.}$	7.8 – 習題 19, 例題 7
9	略	14.1 – 例題 12
10	$\frac{1}{3}\left(\frac{\sqrt{x^2-1}}{x}\right)^3 + C$	7.3 – 習題 11

* 為非勾選習題、勾選習題類似題。
證明題、做圖題過程略過。