

科目名稱: 微積分 (下)(B 群) 開課班級: B 群微積分 考試時間: 4 月 16 日 第二節

I. 填充題. (25 分)

1. If $x \in R$, then the inverse function of $f(x) = 2x^3 + 5$ is $\sqrt[3]{\frac{y-5}{2}}$

2. If $x \in R, x \neq 2$ and $f(x) = \frac{1}{3x+2}$ then $\frac{d}{dx}f^{-1}(x) \big|_{x=\frac{1}{14}}$ is $-\frac{196}{3}$

3. Evaluate $\frac{d}{dx}e^{\sqrt[3]{x^2+1}} = e^{\sqrt[3]{x^2+1}} \frac{2x}{3}(x^2+1)^{-\frac{2}{3}}$

4. If $x > 0$, find $\frac{d}{dx}(x^{\sin x}) = x^{\sin x} \left(\cos x \ln x + \frac{\sin x}{x} \right)$

5. Evaluate $\cos \left(\sin^{-1} \left(\frac{1}{\sqrt{3}} \right) \right) = \frac{\sqrt{2}}{\sqrt{3}}$

II. 計算、證明題. (80 分)

1. If $f(x) = (\cos x + 2)^{\sin x}$. Find $f'(x)$.
2. Prove that $\frac{d}{dx}e^x = e^x$ for $x \in R$.
3. Show that $f(x) = 10 - x^4$ dose not has inverse.
4. Find an equation of the tangent line to the graph of $y = f^{-1}(x)$ at $x = 3$ where $f(x) = x^3 - 24$.
5. Evaluate $\int 7^{3x+1} dx$.
6. Evaluate $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$.
7. Find all extrema of $f(x) = x^2 e^{-2x}$.
8. Evaluate $\int_0^1 \frac{e^x - 1}{e^{2x}} dx$.
9. Evaluate $\int \frac{x^2}{e^{x^3}} dx$.
10. If $f(x) = \log_{\sqrt{e}}(x^2 + 1)$. Find $f'(x)$.

題號	答案	來源
1	$(\cos x + 2)^{\sin x} \left[\cos x \ln(\cos x + 2) - \frac{\sin^2 x}{\cos x + 2} \right]$	6.3 – 習題綜合
2	略	6.3 – 定義
3	略	6.2 – 例題 4
4	$y - 3 = \frac{1}{27}(x - 3)$	6.2 – 例題 8*
5	$\frac{1}{3} \frac{1}{\ln 7} 7^{3x+1} + C$	6.3 – 例題 3*
6	$\ln(e^x + e^{-x}) + C$	6.3 – 定義延伸
7	$f(1) = e^{-2}$ is a local maximum, $f(0) = 0$ is a local minimum.	6.3 – 習題 33
8	$-\frac{1}{e} + \frac{1}{2}e^{-2} + \frac{1}{2}$	6.3 – 習題 24
9	$-\frac{1}{3}e^{-x^3} + C$	6.3 – 習題 15*
10	$\frac{4x}{x^2 + 1}$	6.3 – 習題 42*

證明題、做圖題過程略過, * 為非勾選習題、勾選習題類似題.